Prospective teachers’ beliefs and self-efficacy beliefs about inquiry-based teaching approach in mathematics

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Abstract: The present study focuses on the investigation of prospective teachers’ beliefs and self-efficacy beliefs about the use of the inquiry-based teaching approach in mathematics education during their studies, before and after fieldwork. The aim of the two courses they attended during their studies in a pedagogical department emphasized the understanding of the human involvement on the development of the mathematical concepts through the history of mathematics and the role of investigation and exploration at the teaching of mathematics, as a part of the inquiry-based approach. At the final year of their studies, during the fieldwork they were expected to implement the acquired knowledge about innovative processes in real life classroom situations. The study which conducted with the participation of 73 prospective teachers is divided into three main phases: a) examining their beliefs and self-efficacy beliefs after attending a course about Basic Mathematical Concepts based on the History of Mathematics, b) examining their beliefs and their self-efficacy beliefs after attending a course about the Methodology of Teaching Mathematics in primary education and c) examining the difficulties they face during their first teaching experiences in real life school situations during the last year of their studies. Results indicated that participants seemed to believe in the value of inquiry-based approach and they had high self-efficacy beliefs about using explorations and investigations which were presented at the textbooks; however they had low self-efficacy beliefs about constructing mathematical investigations and explorations by themselves and overcoming teaching difficulties which were related with children’s misunderstandings and time allocation management, during the fieldwork experience.

Keywords: inquiry-based teaching; history of mathematics; beliefs and self-efficacy beliefs

1. Introduction

Higher education programmes in Educational Sciences aim to enable students become acquainted with the necessary mathematical knowledge and with the main teaching strategies and procedures in order to teach mathematics. At the same time those programs aim to create opportunities for the students and prospective teachers to develop positive beliefs and self-efficacy beliefs to teach mathematics in respect to recent guidelines, standards and teaching practices. Inquiry-based teaching and learning is not a recent movement in mathematics education, as it has been recommended as an appropriate basis for student learning in mathematics for the last decades. However numerous studies indicate the difficulties which are faced for its implementation.

Mathematics education aims to enable students to understand mathematics as a creative human endeavour to which they can contribute (Rasmussen & Kwon, 2007). We believe that the introduction of the history of mathematics, by using authentic problems from the history of mathematics, enable students and prospective teachers to understand during
their studies the value of the construction of mathematical concepts and realize the role of exploration, investigation and experimentation on the development of the science of Mathematics (Panaoura, 2016).

Many curricula have been designed to promote higher-order thinking, inquiry-based learning, or have been changed during a reform, but one of the greatest factors in improving children’s academic achievements is the quality of the teacher (Marshall & Horton, 2011). Two of the influential factors in any curriculum reform are teachers’ preparation through higher education programs and teachers’ beliefs about change (Gooya, 2007). The challenge for higher education and especially the departments of educational studies is to facilitate their students, who are at the same time prospective teachers, to adopt the values of the inquiry-based pedagogy and to be able them to regulate their teaching behaviour in order to face teaching unexpected situations and overcome them fluently and flexibly.

The present study investigates the effectiveness of a higher education program of a pedagogical department based on (i) understanding the basic mathematical concepts through the use of the history of mathematics, (ii) understanding the recent pedagogical practices, which emphasize the processes of exploration and investigation in mathematics education and (iii) having experiences in teaching mathematics at the last year of their studies, during the compulsory fieldwork in primary school education.

More specifically, the aim of the present study was threefold:

1. To investigate the impact of the program on prospective teachers’ beliefs and self-efficacy beliefs about the nature of mathematics by using the history of mathematics under an inquiry-based teaching perspective
2. To investigate the impact of the program on prospective teachers’ beliefs and self-efficacy beliefs about the use of inquiry-based approach in the teaching of mathematics
3. To identify the teaching practices they follow, during the fieldwork experience in primary school, which are related with the inquiry-based approach.

2. Theoretical framework

2.1 Inquiry-based teaching approach

Children from the day of their birth explore their environments through inquiry, and this natural tendency has to be transferred in education in order to engage students in active and meaningful learning activities (Wang, Kinzie, McGuire & Pan, 2010). Inquiry-based teaching and learning is based on the principles of social constructivism (Alluls & Shore, 2008), according to which learner assimilates a new situation and experience by building on the previous ones. It has also its origins in the processes of the scientific inquiry (Song & Looi, 2011), e.g. choosing, gathering and analysing data in respect to posed questions, constructing and presenting explanations by engaging in investigations, after conducting the appropriate experiments (Krajeck & Blumenfeld, 2006). According to Chapman (2011) inquiry-based teaching allows students’ questions and curiosities to drive the activities, make uses of multiple ways of knowing, relate previous experiences and current situation and allows for creation of new perspectives.

Many studies on the specific domain have been developed at the higher level of education. Kogan and Laursen (2014) examined undergraduates’ course taking inquiry-based experience in college mathematics. They engage students in exploring mathematical problems, propose and test conjectures, develop proofs and explain their ideas. In addition inquiry -based teaching is characterized by a certain amount of experimental procedures and self-regulated leaning sequences where student autonomy is emphasized (Ropohl, Ronnebeck, Bernholt & Koller, 2013). Four components are presented at the inquiry instructional model by Marshall and Horton (2011): engage (when misconceptions and prior knowledge are exposed), explore (when a learner in investigates a concept), explain (when a prior knowledge is used in order to generate conceptual understanding) and extend (when the knowledge is applied to new situations). Research in inquiry activity shows that lower track students can enhance their argumentation (Tuan, Chin, Tsai & Cheng, 2005), while high – ability students enhance their inquiry skills in conducting inquiry research (Bell, Smetana & Binns, 2005). At the same time inquired-based procedures permit teachers to have deeper insight into their students’ thinking and have a better understanding of their misconceptions and misunderstandings (Burns, 2004).
Beliefs about mathematics and how to teach it are influenced by experiences with schooling (Gates, 2006). Chin and Lin (2013) claim that teachers face obstacles and problems such as: (i) they do not experience inquiry-based learning in mathematics during their own school years, which affect their beliefs about the necessity of the approach (ii) they do not have complete understanding of the inquiry-based teaching, which influences their self-efficacy beliefs on using it (iii) there are practical constraints such as the allocated teaching hours not being enough, and (iv) the influence of the teaching for success in tests. Based on the idea that one of the most important factors applying innovative procedures is the teachers, the present study examines their beliefs about inquiry-based learning and especially the use of the processes of exploration and investigation for the teaching of mathematics, their self-efficacy beliefs in using them and their teaching practices during the respective expected implementation.

According to Taylor and Bilbrey (2011) the research outlines two facets of inquiry-based instruction which are open education and differentiation. The major characteristic of the open education is that instruction is driven by the desires of the pupils, while the differentiation approach allows pupils’ preferences to guide how particular content is encountered. The learner-focused perspectives of mathematics education requires teachers to use pedagogical methods which actively engage pupils in developing conceptual understanding of mathematical concepts (Chapman, 2011). The teacher’s role has evolved from concept deliverer to concept facilitator. In order to be able to leave the pupils’ curiosity to drive the teaching process, teachers need strategies of flexibility and self-management. A presupposition for an inquiry-based approach is the teachers’ personal experience of inquiry-based approach (Soprano & Yang, 2013). A possible challenge is however that many teachers teach the ways they were taught and of them many were taught from a traditional approach (Malloy, 2003). This traditional approach affected directly or indirectly their beliefs. Wang and Lin (2008) claim that the only way to improve teachers’ thinking and their respective practice in teaching is to involve them in the self-reflecting process. Without experience in inquiry-based learning in teacher preparation programs, teachers either omit inquiry-based teaching or rely on professional development programs (Lebak & Tinsley, 2010). The study of Soprano and Yang (2013) about the inquiry teaching in science indicated that although it is important for teachers to be comfortable with this approach, providing pre-service teachers with the opportunity to engage in inquiry based practices contribute to deep understanding of inquiry – based instruction and at the same time increased their self-efficacy for teaching the specific subject.

### 2.2 The use of the history of Mathematics in Higher Education

The idea of using the history of mathematics in education is not new (Goktepe & Ozdemir, 2013). Over the past three decades researchers from various countries have discussed the possibility of introducing new concepts within relevant historical context (Yee & Chapman, 2010), at different educational levels. Since 2009, the European Conference for Research in Mathematics Education (CERME) has a working group which discusses the historical roots of mathematical concepts, theoretical issues about the role of the history of mathematics and studies which are conducted at different levels of education.

There are several reasons to incorporate the use of the history of mathematics in education, and the major one is the impact of such a practice on the development of the mathematical disposition of students (Clark, 2006). Using authentic problems from the history of mathematics provides materials for students to actively engage in classroom discourse (Gulikers & Blom, 2001), and to realize the role of the construction of the science of mathematics. Studying the development of mathematical ideas also opens up the possibility of seeing mathematics as a socio-cultural creation and helps “humanize” mathematics (Fauvel, 1991). A journey through the history of mathematics enables learners to construct mathematical meanings and support new conceptions about mathematics by changing their existing beliefs and attitudes (Dubey & Singh, 2013).

Historical and epistemological analysis of the construction of mathematical concepts helps teachers understand why a certain concept is difficult for students to grasp. Such an understanding is important, because it can inform selection of tasks/problems to introduce a particular concept, the strategies teachers employ in helping students develop understanding of this concept, and the time they allot to working on this concept (Barbin et al., 2000). Gagatsis and
Panaoura (2014) underline the valuable role of the historical approach in mathematics teaching and learning, by examining the concept of absolute value, as it helps students to understand that mistakes, doubts, experimentations, arguments are an integral part of mathematics in the making. The historical study of a concept is used in order to understand the students’ probable difficulties and misconceptions. Brousseau (1997) studied the historical development of the concept of limit in order to identify the students’ epistemological obstacles.

Some researches describe the affective impact from using the history of mathematics in education (e.g. Furinghetti, 2007; Marshall, 2000) and others discuss the necessity to include the history of mathematics in teachers’ university programs (e.g. Fleener, Reeder, Young & Reylands, 2002) in order to train teachers to use it with their students. Teachers have long been encouraged through curriculum and the scientific community in mathematics to incorporate aspects from the history of mathematics into their teaching (Lopez-Real, 2004), however they face difficulties which are related with their knowledge, with their students’ misconceptions and negative attitudes and the teaching hours limitations (Siu, 1997). The mathematics teachers in the study by Lit, Siu and Wong (2001) were very supportive in theory for using history in their teaching. Siu in an invited talk given at the working conference of the 10th ICMI study on the role of mathematics in mathematics education, offered a list of thirteen reasons why a school teacher hesitates to make use of the history of mathematics in classroom teaching such as “I have no time for it in class”, “Students don’t like it”, “There is a lack of teacher training on it”, “Students do not have enough general knowledge on culture to appreciate it”, etc. The suggestions which are included in Curriculum or Reports of Committees do not necessarily mean that teachers are able to apply them in their teaching, either due to their lack of positive beliefs and self-efficacy beliefs or due to teaching difficulties and obstacles, which they are unable to overcome when they face them.

2.3 Beliefs and self-efficacy beliefs

Teachers’ beliefs and their respective self-efficacy beliefs are key informants of their instructional approaches and their conceptions regarding the difficulties and limitations. Beliefs as a part of the affective domain (Goldin, 2002) construct a multifaceted concept which in the specific case includes their beliefs about themselves, the nature of the discipline of mathematics and the factors which affect the learning of mathematics (Charalambous, Philippou & Kyriakides, 2008). According to Song and Looi (2011) much research has shown that there is a strong correlation between teachers’ beliefs, their self-efficacy beliefs and practices. The construct of self-efficacy beliefs constitutes a key component in Bandura’s social cognitive theory according to which it is consisted of his/her self-image about the respective ability to achieve a goal. Bates, Kim and Latham (2011) indicated that pre-service teachers’ mathematical performance is related to their mathematics self-efficacy and their mathematics teaching efficacy. Teachers who are very confident in their ability to teach mathematics they believe that they can have a positive effect on their students’ behaviour and performance.

Educational reforms impose new demands to the work of teachers. Van den Berg and Ros (1999) found that when a reform is introduced teachers express self-concerns and after it is established they express concerns about its impact on students. It is important to examine the changes in teachers’ concerns, beliefs and practices which arise as a result of an innovation (Tunks & Weller, 2009). The study of Charalambous and Philippou (2010) focused on teachers’ concerns and efficacy beliefs toward curriculum reform. In order to maximise the effectiveness of a curriculum reform they suggested informing teachers further and provide them with systematic and substantial guidance and support.

As Crawford (2007) claims, the teachers’ beliefs impact on their decisions in interpreting and applying the curriculum by scheduling the specific lesson plans. A study by Song and Looi (2011) investigated how teachers with different beliefs enacted the same mathematics lesson on inquiry principles and examined the pupils’ progressive inquiry process and outcomes. Results indicated that many obstacles arise in teaching with inquiry because it requires unfamiliar skills for the traditional mathematical classrooms.

The emphasis is on finding out how can we best educate or train pre- and in-service teachers in order to provide support for student-centered ways of teaching. At the same time in many countries, according to Dorrier and Garcia
(2013), school teachers must be offered sufficient support in order to overcome their general disinclination towards mathematics. Kazempour (2009) found that inquiry-based professional development opportunities for teachers were an important contributing factor for the successful implementation of inquiry-based instruction. As Taylor and Bilbrey (2011) claim inquiry-based instruction is shown to spur greater teacher self-efficacy which leads to more positive perceptions of inquiry-based learning, while teachers who believe in their own abilities are more likely to engage in open communication. Emphasis is given at the university programs of studies for pre-service teachers. Inquiry–based instruction in this case has shown to affect positively the future self-efficacy of teachers, subsequently impacting their later choices to utilize the same approach (Alsup, 2005). At the same time research shows that teachers with limited knowledge of scientific inquiry may lack the required level of self-efficacy to teach it effectively (Northcutt & Schqartz, 2013). Teachers who during their studies or at least during in–service training programs develop a high level of efficacy on planning and applying inquired based instruction are more willing to take on new challenges (Gurvitch & Metzler, 2009), such as using technology as an inquiry tool for investigation and exploration.

3. Methodology

3.1 Sample

Participants at the present study were 73 (18 males and 55 females) students who were studying at a four year bachelor program in Educational Studies at a private university in Cyprus. Their aim was to become primary school teachers. The ratio of females to males at the occupation of teachers at Cyprus is almost 60% to 40%. At the first phase of the study, during the first year of their studies, the participants were 76; however 3 of them decided to change the programs of their studies, so they were excluded from the sample. Four participants were accepted to take part voluntarily at a half-time semi-structure interview at the final stage of the study.

3.2 Research Instruments and Procedure

To examine participants’ beliefs about the nature of mathematics a questionnaire (items are presented at Table 1) was constructed based on the study of Charalambous, Panaoura and Philippou (2009). To examine participants’ beliefs and self-efficacy beliefs about using inquiry-based teaching approach in the specific context of Cyprus a second questionnaire was constructed. The questionnaire (items are presented at Table 2) was based on the study of Charalambous and Philippou (2010) about the previous implementation of a mathematics curriculum reform in Cyprus. Firstly four in-service teachers were asked to complete both the questionnaires in order to judge the clarity of the items. The final versions of each questionnaire were consisted of 18 and 15 items respectively. A 5-point Likert scale ranging from 1=strongly disagree to 5=strongly agree was used.

In respect to the procedure, participants completed the two questionnaires four consecutive times: (i) at the beginning of the 1st semester of their studies, before attending the course “Basic mathematical concepts”, (ii) at the end of the 1st semester, after the final exams of the course, (iii) at the beginning of the 5th semester of their studies, before attending the course “Methodology for the teaching of mathematics” (iv) after the final exams of the course.

The mathematics teacher preparatory program considered in this study has been designed and implemented at the University since 2007. The program consists of a mathematics content course and a teaching methodology course. The first one is during the first year of studies (1st semester) and the second one during the third year of studies (5th semester). Each course was taught as a one semester, 6 ects course that lasted 13 weeks. During each of the 13 weeks, students were expected to attend two one hour lecture delivered by the author and an hour of a group work in small groups, where they had to solve problems, they discussed, they presented their work and they investigated the mathematical concepts. The first course consisted of units about the historical development of mathematical concepts such as: (i) the structure and the evolution of number systems (Egyptians, Babylonians, Greek), (ii) the main operations and algorithms (Egyptians, Indians) and the role of zero in different ancient arithmetic systems, (iii) Euclidean geometry
and mathematical proof (definition, axioms, theorems), (iv) the highest common factor by using the Euclid’s method and the minimum common multiple in problem solving and (v) combinatorial problems and the Pascal’s triangle. The second course consisted of units about (i) the basic trends, emphases, standards and practices for the teaching of mathematics at primary education, (ii) the cognitive and developmental theories on the construction of the procedural and conceptual understanding (iii) the differentiation in education, based on the learning styles and the cognitive styles, (iv) the role of different representations on the understanding of mathematics, (v) the inquiry-based teaching and learning approach (the role of investigation and exploration in relation to problem solving and experimentation), (vi) the use of technology and (vii) the special teaching methods for the basic mathematical units (numbers and operations, geometry and spatial ability, statistics and probabilities, measurement and algebraic thinking).

At the final year of their studies (8th semester) they had a course called “School Experience”, during which they had to teach for 8 weeks at primary education school. With the guidance of a supervisor they had to teach all the school lessons, however at the present study we concentrate our attention only in mathematics. They had to schedule and implement 16 lessons in mathematics. Four participants took place at an individual semi-structured interview which was conducted based on the following up dimensions: (i) the use of inquiry-based approach (through the curriculum and the textbooks), (ii) the construction of explorations and investigations by themselves, (iii) the difficulties they face during the teaching, and (iv) the strengths and limitations of using the inquiry-based approach and (v) the use of the history of mathematics as part of the inquiry-based approach in primary school mathematics.

3.3 Ethics Statement

Participants were informed at the first meeting of the first course about the aims of the study, the expected contribution by themselves and their right to interrupt their participation whenever they wanted. They signed for their acceptance to participate voluntarily and they received a signed declaration by the academic that the participation could not be related by any way with the grading of the course.

4. Results

The presentation of the results is based on the three main research questions. Firstly we concentrated our attention on prospective teachers’ beliefs and self-efficacy beliefs about the nature of mathematics and the impact of an intervention course based on the history of mathematics. Then we present the results about the role of the inquiry-based approach on mathematics teaching and learning and finally we concentrate our attention on the qualitative data derived by prospective teachers’ fieldwork experience, concerning the practices they followed and the difficulties they faced during the implementation of the inquiry-based teaching approach.

4.1 Beliefs about the nature of mathematics

In order to examine whether there were changes in prospective teachers’ beliefs about the nature of mathematics we first subjected their responses to exploratory factor analysis (KMO=0.718, p<0.01). The analysis of the data resulted in three factors with eigenvalues greater than 1 for all the four measurements which explained the 67.3%-71.2% of the total variance (for the four measurements). As presented in Table 1, the 18 items were consistently clustered in the same factors for all the measurements. The items at the first factor expressed a formalistic perspective of mathematics based on rules, symbols and algorithms. The second factor corresponded to an experimental view of mathematics. Finally the items of the third factor expressed their beliefs about the role of the person on the learning of mathematics.

<table>
<thead>
<tr>
<th>Item</th>
<th>Factor1</th>
<th>Factor2</th>
<th>Factor3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics is a set of algorithms and procedures.</td>
<td>.86 (.73) [.77] {.71}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mathematics is a set of symbols.</td>
<td>.84 (.71) [.57] {.64}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mathematics is mainly a set of rules and</td>
<td>.74 (.70) [.57] {.71}</td>
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Mathematics is a domain with precise results. .75 (.74) [.64] [.69]
Mathematical proofs are written in a formal way. .63 (.57) [.56] [.57]
Mathematical definitions are strict. .71 (.75) [.73] [.65]
Each mathematical problem has a single mathematical solution. .68 (.69) [.55] [.73]
The mathematical sentences are always true. .83 (.75) [.64] [.71]
Mathematics is a dynamic science with changes though the centuries. 84 (.68) [.81] [.74]
Mathematics serves certain human needs. .83 (.65) [.69] [.58]
Mathematics is important because it is useful for human beings. .65 (.73) [.78] [.79]
The development of mathematics is associated with certain human needs and problems. .71 (.62) [.61] [.52]
Mathematics is an evolving body of knowledge necessary for daily activities. 74 (.69) [.79] [.66]
One must be talented to learn mathematics. .78 (.78) [.53] [.57]
A student’s success in math depends more on genetic factors than on teaching. .64 (.55) [.71] [.58]
No matter how hard a teacher tries, there are students who will not understand mathematical concepts and procedures. .68 (.73) [.74] [.71]
All the students are able to learn how to solve mathematical exercises. .74 (.69) [.62] [.63]
Some students are not able to learn how to solve mathematical problems. .72 (.68) [.69] [.68]

*The loadings for the second, third, and fourth measurements are presented in parentheses, squared brackets, and curved brackets, correspondingly

F1= formalistic perspective, F2= experimental perspective, F3=individual responsibility for learning

Table 1. Factor loadings of the three factors against the items associated with participants’ beliefs about the nature of mathematics

We used the grouping of the items of the three factors to trace the evolution of beliefs about the nature of mathematics. We examined the development of the participants’ beliefs about the nature of mathematics and the learning of mathematics by applying analysis of variance of the three factors at the four measurements (Figure 1).

Figure 1. The evolution of prospective teachers’ beliefs about the nature of mathematics
The formalistic beliefs about the nature of mathematics were declined after the attendance of the second course (4th measurement), while the beliefs about the experimental perspective of the nature of mathematics were increased gradually. Specifically, participants’ beliefs with a formalistic perspective were significantly stronger than those they held at the 4th measurement ($t_{1,4} = 8.47$, $p<0.01$). The slight increase in the strength of their respective beliefs between the 2nd and the 3rd measurements was not significant ($t=1.52$, $p>0.05$). At the same time the gradually increase of the experimental perspective of mathematics was statistically significant between the 4th measurement and all the previous measurements ($t_{1,4} = 7.63$ $p<0.01$, $t_{2,4} = 5.15$ $p<0.01$, $t_{3,4}=6.08$ $p<0.01$). However participants’ beliefs about the ability of all people to learn mathematics were intensified negatively (all the items were firstly converted positively). They seemed to insist in believing that there are people who are not talented in mathematics and they cannot overcome their difficulties by effort.

### 4.2 Beliefs and self-efficacy beliefs about the inquiry-based teaching approach

Exploratory factor analysis was used in order to identify the dimensions of the questionnaire in respect to the participants’ responses, concerning their beliefs and self-efficacy beliefs about the use of the inquiry-based teaching approach. The principal component analysis of participants’ responses to the items of the questionnaire revealed three factors (KMO= 0.873, $p<0.01$) with eigenvalues greater than one (Table 2).

The first factor was consisted of 6 items, expressing the participants’ self-efficacy beliefs concerning the management of time and situations during teaching by using inquiry-based approach. The second factor consisted of 5 items, expressed their beliefs about teaching practices in the implementation of the inquiry-based approach. Finally the third factor was about their acquired knowledge about the use of the specific approach, and it consisted of 4 items.

<table>
<thead>
<tr>
<th>Item</th>
<th>Factor1</th>
<th>Factor2</th>
<th>Factor3</th>
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<tbody>
<tr>
<td>I am able to organize pupils work in peers during an investigation.</td>
<td>.83 (.78) [.77] [.68]</td>
<td></td>
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</tr>
<tr>
<td>I am able to make pupils feel free to use imagination in mathematics.</td>
<td>.82 (.73) [.75] [.84]</td>
<td></td>
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</tr>
<tr>
<td>I am able to make pupils feel free to use manipulative in mathematics whenever the need them.</td>
<td>.77 (.79) [.82] [.77]</td>
<td></td>
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</tr>
<tr>
<td>I am able to use the exploration proposed by the mathematics textbooks.</td>
<td>.81 (.75) [.68] [.73]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I am able to use the investigation proposed by the mathematics textbooks.</td>
<td>.63 (.67) [.66] [.77]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I am able to organize the time for an investigation.</td>
<td>.74 (.75) [.54] [.67]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I am able to make pupils communicate in mathematics during exploration tasks.</td>
<td>.68 (.62) [.58] [.63]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teachers need to organize the time for an investigation or an exploration.</td>
<td></td>
<td>.81 (.75) [.81] [.64]</td>
<td></td>
</tr>
<tr>
<td>Pupils communicate in mathematics during investigation tasks.</td>
<td></td>
<td>.83 (.76) [.73] [.62]</td>
<td></td>
</tr>
<tr>
<td>Teachers have to propose their own exploration of a mathematical concept.</td>
<td></td>
<td>.75 (.71) [.62] [.77]</td>
<td></td>
</tr>
<tr>
<td>Teachers are able to use the investigations and explorations proposed at the textbooks.</td>
<td></td>
<td>.71 (.72) [.79] [.62]</td>
<td></td>
</tr>
<tr>
<td>It is useful to ask pupils to compare what</td>
<td></td>
<td>.78 (.69) [.63] [.81]</td>
<td></td>
</tr>
</tbody>
</table>
It is useful to encourage pupils to make different plans to accomplish a problem solving task.  

At the inquiry-based learning there is not any time limitation.  

In order to plan an exploration teacher has to take into account pupils’ interests.  

It is useful to ask pupils to explain the results they find.

*The loadings for the second, third, and fourth measurements are presented in parentheses, squared brackets, and curved brackets, correspondingly

F1 = Self-efficacy beliefs, F2 = beliefs, F3 = knowledge

Table 2. Factor loadings of the three factors against the items associated with participants’ beliefs and self-efficacy beliefs about the inquiry-based teaching in mathematics

Results which are presented at Figure 2 indicated that participants at the initial measurement had extremely high self-efficacy beliefs about their abilities to manage the teaching situations while using the inquiry-based approach (3.22). Those beliefs remained high after the attendance of the course on basic mathematical concepts (3.31 and 3.29 at the 2nd and 3rd measurement respectively). However the attendance of the course on the teaching of mathematics led to the decline of those beliefs (2.88), as they probably realize the difficulties which derived from the implementation of the inquiry-based teaching processes. The difference on those self-efficacy beliefs was statistically significant between the 4th measurement with all the previous measurements ($t_{3,4} = 2.23$, $t_{2,4} = 2.41$, $t_{3,4} = 2.52$, p < 0.05).

Figure 2. The evolution of prospective teachers’ beliefs and self-efficacy beliefs about the inquiry-based teaching approach in mathematics

In respect to the participants’ beliefs about the teaching practices in the implementation of the inquiry-based approach, results indicated that they had positive beliefs at the initial measurements (2.95 and 2.88 at the 1st and 2nd respectively) which became slightly stronger at the 3rd measurement (3.02). Although they had not yet attended the course on the methodology about the teaching of mathematics, they had attended during the four semesters of their studies relevant courses about the general teaching methodology, about cognitive and developmental psychology and about the teaching of sciences. Their beliefs about the use of inquiry-based approach in the teaching of mathematics became stronger after attending the course about the teaching of mathematics (3.42). Similar was the results in the case of the third factor which expressed participants’ knowledge about the use of the inquiry-based approach, with positive initial situation (2.84) and stronger beliefs at the end (3.48).

In sum, participants hosted more experimental beliefs at the end of the compulsory courses and before the fieldwork at primary school compared to those the harboured at the beginning of their formalistic beliefs about the nature of the mathematical knowledge were less intense at the end of the program. Regarding their beliefs about
person’s effort and influence on the learning of mathematics their beliefs did not change significantly over the courses. At the same time the course succeed in improving participants’ beliefs about the role of the inquiry-based approach in mathematics and added the expected knowledge about the inquiry-based teaching processes. However participants’ self-efficacy beliefs in implementing those processes declined.

4.3 Participants’ practices and their beliefs about the implementation of the inquiry-based teaching approach

The semi-structured interviews with four participants enabled us to concentrate our attention more qualitatively on the relations of their beliefs with the practices they followed in order to use the inquiry-based approach at the teaching of mathematics during fieldwork. After coding their responses in respect to the five dimensions of the interview, it seems that participants recognized the value of using the inquiry-based approach. However they seemed to lack confidence to propose their own investigations and explorations. They did not have high self-efficacy beliefs about the development of appropriate activities for pupils with different learning achievements.

“I used the examples which were presented at the textbook in order to be sure that pupils would understand what they were expected to do”

“I do not think that the teacher would allow me to do something else than following the guidelines which were presented at the textbook. All the days I was at the specific class I do not remember her to use her own exploration or investigation”

“It was difficult for me to use even the examples which were presented at the textbooks…I did not feel confident and I understood that my teaching methods depended only on the pupils with high performance in mathematics. I did not know how to facilitate those with low performance”

They underlined many difficulties they faced which disclose their lack of confidence on implementing inquiry-based approach.

“It is interesting to include a mathematical exploration or investigation, however it is difficult to prevent pupils to discuss ideas which are not related with the specific mathematical concept you are trying to teach…Teacher needs time and it is difficult when you are student and you have to follow the curriculum guidelines and the teacher’s expectations.”

“When I had decided to use an exploration which was presented at the textbook of the 3rd grade, I spent almost 30 minutes to discuss with them. It was a traumatic experience as the teacher interrupted me and asked them to solve an exercise from the textbook.”

Although there were asked to pose the strengths and the limitations of using the inquiry based approach, they underlined mainly the limitations, as there were derived from the difficulties they had faced. A participant expressed the opinion that pupils were too young to let them explore a mathematical concept individually and for this reason he preferred the guided introduction of a concept. Another participant claimed that even in the upper grades of primary education “it is easier to guide them in order to be sure rather than to be lucky to discover something. What if nobody derives to the expected?”

“Inquiry-based approach is useful, especially for pupils with low performance in mathematics as it reveals their misunderstandings and misconceptions… It is difficult to find the way to face those misunderstandings”

“It is impossible to work at the same time with all the pupils during an investigation. Pupils with high performance solve the tasks easily, while the students with the low performance are not able to understand and hesitate to express their thoughts”.

They expressed positive feelings about their experiences to acquire knowledge about the history of mathematics. “I never thought that the human needs throughout the centuries developed the science of mathematics”, “I thought that mathematics was stable and the same for all the cultures”. At the same time they did not feel confident to use the history of mathematics as part of the inquiry-based approach in primary school mathematics. “It could be useful in secondary education, especially in geometry; it is a waste of time in primary education as pupils are not able to realize the value of
the concept evolution”. A participant added “it was difficult for me to understand all those historical concepts during the course, I was stressed and I needed more time to understand and solve the tasks. Only the extra time during the teacher’s office hours enabled me to pass the final exams. It is impossible to spend analogous time with pupils at primary school teaching”. Most of them did not have the opportunity to teach a concept by using a historical perspective, although there are few examples at the textbooks. Only a student had the experience to be able to teach the place value at the 3rd grade by using an investigation presented at the textbook where an archaeologist found the presented numbers on stones which belong to ancient Crete. “It was difficult for me to explain them that they did not have to use in future those symbols…It was time consuming and the teacher insisted not using this investigation. I used it only as part of an exercise”.

5. Conclusions and Discussion

International councils for the teaching of mathematics will contribute to call for inquiry-based teaching approach in mathematics. The current study examined prospective teachers’ beliefs and self-efficacy beliefs about using the inquiry-based approach in mathematics. A university program, based on two courses (about the basic mathematical concepts through the use of history and about the teaching of mathematics), aimed to enable them to teach mathematics by following the recent trends and practices. Analogous beliefs, conceptions and difficulties in different ranges and degrees probably, are expected to be faced in different educational systems which try to introduce mathematical concepts by using an inquiry-based approach or more generally which are proposing innovations for the teaching of mathematics.

The high means of participants’ beliefs indicated the positive disposition about using the inquiry-based teaching approach in mathematics. However they need positive experiences in order to develop high self-efficacy beliefs about the practices they are using. The uncertainty about the teaching results for all the pupils confirmed previous results of studies when innovations is introduced in education and there is a lack of personal teaching experiences (Tunks & Weller, 2009).

The sample of the participants seemed to have adequate knowledge about the basics concerning the inquiry-based teaching approach; however they did not have high self-efficacy beliefs in implementing it. The current study provided evidence that although prospective teachers have positive beliefs about the value of the inquiry-based teaching approach, they do not apply its features into their teaching practice during the fieldwork satisfactorily. They do not transfer any positive experiences as probably the teacher-centred learning experiences they had for years in secondary education were too strong, and they feel more confident to teach by the way they were taught. A following up study has to examine the impact of their performance on mathematics during their university studies on those beliefs and self-efficacy beliefs.

Undoubtedly the university courses have to take into consideration as well the participants’ inter-individual differences. Probably less subject matter needed to be covered in order to allow for better digestion of the ideas considered in the courses. Their comments suggested that certain aspects of the content we had envisioned to improve their beliefs about the role of the history of mathematics at an inquiry-based teaching approach were not perceived as such by the study participants. Their experiences during the first course were not enough in order to transfer by their own those ideas in the teaching of mathematics. Additionally the use of the history of mathematics did not facilitate all the prospective teachers start seeing mathematics differently as they had difficulties to understand the complex problems and the respective great mathematicians’ arguments.

Emphasis has to be given on studying further prospective teachers’ difficulties in implementing the inquiry-based teaching approach in mathematics. Teachers who “accommodate” prospective teachers at their classrooms seemed to prevent them from trying to implement innovative processes. Prospective teachers need time and “space” for experimentation on using inquiry-based approach. A future study has to relate the beliefs and practices of both the abovementioned groups.
References