Question Kastner Formulae of Circular Tunnel

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Abstract: Stress and deformation of surrounding rock are basic issues in tunnel engineering, directly affecting the safety evaluation and the support or lining measures to a tunnel. Kastner formulae proposed in 1950s describe the plastic zone range of a circular tunnel and the stress distribution in plastic zone, and have served as the theoretical basis of tunnel analysis in past decades. Supposing the surrounding rock is ideal elastic-plastic material with Mohr-Coulomb or Mises criterion, we proved that the support process to a circular tunnel is an unloading process in terms of plastic mechanics. The conclusion was verified by FLAC calculation. Both the stress increment and the displacement increment of surrounding rock in support stage are consistent with the elasticity theory. Therefore, Kastner formulae are only applicable to the excavation stage of tunnel, not to the support stage.

Keywords: Tunnel; Kastner Formula; Loading Criterion; Plasticity Analysis

Introduction

An important problem in tunnel engineering is to determine the plastic zone range and supporting stress of surrounding rock. The kastner (H. Kastner) formula for circular tunnels put forward in the 1950s and 1950s describes the stress distribution in the plastic zone of the tunnel and the lining support. The relationship between resistance and radius of plastic zone of surrounding rock[1], see expressions (1) and (2):

$$\begin{cases} \sigma_r = c\cot\varphi - (p_i + c\cot\varphi)(\frac{r}{a})^{\frac{2\sin\varphi}{1-\sin\varphi}} \\ \sigma_\theta = c\cot\varphi - (p_i + c\cot\varphi)(\frac{r}{a})^{\frac{2\sin\varphi}{1-\sin\varphi}} \dfrac{1+\sin\varphi}{1-\sin\varphi} \end{cases} \quad (1)$$

$$R_p = R_0\left(\frac{(p_o + c\cot\varphi)(1-\sin\varphi)}{p_i + c\cot\varphi}\right)^{\frac{1-\sin\varphi}{2\sin\varphi}} \quad (2)$$

In the formula, the stress is positive in tension and negative in pressure, PI is tunnel internal pressure, P O is external pressure (i.e., ground stress); R0 is the tunnel radius, C is Rock cohesion, φ and internal friction angle. The kastner formula is also called the modified Finer formula (hereinafter referred to as Karnofsky formula). It is currently used in tunnel mechanics analysis the basic formula of is the theoretical basis for tunnel design and construction and has been written into textbooks of rock mechanics and tunnel mechanics analysis[2,3]. The applicable conditions of the formula are: the surrounding rock is an ideal elastic-plastic material, the initial ground stress is isotropic, and the yield criterion is mo according to the Coulomb criterion, the stress on the tunnel is a plane strain problem.

According to the equilibrium differential equation and yield criterion of the surrounding rock element, as well

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as the boundary conditions of the elastic-plastic zone, it can be obtained that the stress distribution formula of tunnel surrounding rock, and then Karnofsky formula is obtained. The whole derivation process of Karnofsky's formula does not involve the material's constitution. Since Karl Fischer's formula was put forward 60 and Hou gongyu has questioned Karl fischer's formula over the past years,[4] and pointed out the simultaneous action of support reaction and ground stress in the formula and " The actual situation is inconsistent. Since the plastic mechanical behavior is related to the loading path, in-situ stress and support stress are loaded simultaneously and sequentially. The results are not the same. Hou Gongyu and Niu Xiaosong Based on Plastic Mechanics Levy-Mises Incremental Constitutive Relation, in Under the conditions of P yield criterion and Hoek-Brown yield criterion, the ideal elastoplastic solution 5,6 for axisymmetric circular tunnel is derived. However, they did not consider the loading and unloading criteria. The derivation process is not essentially different from Karnofsky's method, except that the Mohr-Coulomb (M-C) yield criterion in Karnofskys method is replaced by the D-P criterion or Hoek-Brown criterion. The stress and displacement curves of surrounding rock obtained by Wen,[5] are obviously different from those obtained by FLAC3D and calculation results (see Wen,[5] Zhong Tu ,10 ,He Tu and 11).

In this paper, the loading and unloading of tunnel support steps are judged, and the finite difference numerical calculation software FLAC2D is used to analyze the influence of excavation and support on the stress distribution of circular tunnel, so as to illustrate the correct method of tunnel stress analysis.

1. Loading and unloading judgment

According to the theory of plastic mechanics, for ideal elastic-plastic materials, the subsequent yield surface coincides with the initial yield surface. If yield function for and F, there are the following loading and unloading criteria:

$$\begin{cases} f<0, & \text{Resilience} \\ f=0, \quad df=\dfrac{\partial f}{\partial \sigma_{ij}}d\sigma_{ij}=0, & \text{Loading} \\ f=0, \quad df=\dfrac{\partial f}{\partial \sigma_{ij}}d\sigma_{ij}<0, & \text{Unloading} \end{cases} \quad (3)$$

According to elastic mechanics, the stress distribution of cylinder in elastic state is:

$$\sigma_r=-\frac{\frac{b^2}{r^2}-1}{\frac{b^2}{a^2}-1}q_a-\frac{1-\frac{a^2}{r^2}}{1-\frac{a^2}{b^2}}q_b, \quad \sigma_\theta=\frac{\frac{b^2}{r^2}+1}{\frac{b^2}{a^2}-1}q_a-\frac{1+\frac{a^2}{r^2}}{1-\frac{a^2}{b^2}}q_b \quad (4)$$

2. Mechanical interpretation of tunnel excavation and support process

The conclusion in the previous section that excavation is unloading is inconsistent with people's traditional understanding for a long time. The following is a mechanical analysis of tunnel excavation and the supporting process. No matter whether excavation is carried out or not, we can always regard the surrounding rock of the tunnel as a cylinder whose outer diameter is far greater than the inner diameter, except in different areas The internal pressure of the cylinder is different at the stage. If the tunnel is in uniform ground stress, i.e. the lateral pressure coefficient is 1, then the circle before excavation. The internal pressure of the cylinder is the same as the external pressure, and the stress in the surrounding rock is equal everywhere. Excavation means that the internal pressure of the cylinder is withdrawn and added according to plastic mechanics. Unloading criteria, see (3) formula, excavation is a loading process, so plastic zone appears in surrounding rock. When lining support is carried out, then the surrounding rock will be in the unloading process described by plastic mechanics, which will make the original plastic zone become the residual deformation zone. After support stress and displacement of surrounding rock are equal to the value before support minus the elastic stress or displacement caused by support.

3. Numerical calculation verification

3.1 Calculation model and scheme

Due to the symmetry of the model, take 1/4 and cylinder for analysis. Use finite difference method software FLAC 5.0 for analysis and calculation. The tunnel radius is 6 meters and the in-situ stress is 20MPa. The material parameters are: density 2500kg/m^3, elastic modulus 6.78GPa, Poisson's ratio 0.21, cohesion 0.8MPa, tensile strength 2.0MPa, internal friction angle 30, and dilatancy angle 0. Using Ideal Elasto-Plastic Model and Moore-Cullen Yield Criterion. The construction process is: first excavate the tunnel, then apply 7MPa lining stress. Use FLAC and Simulate this process.

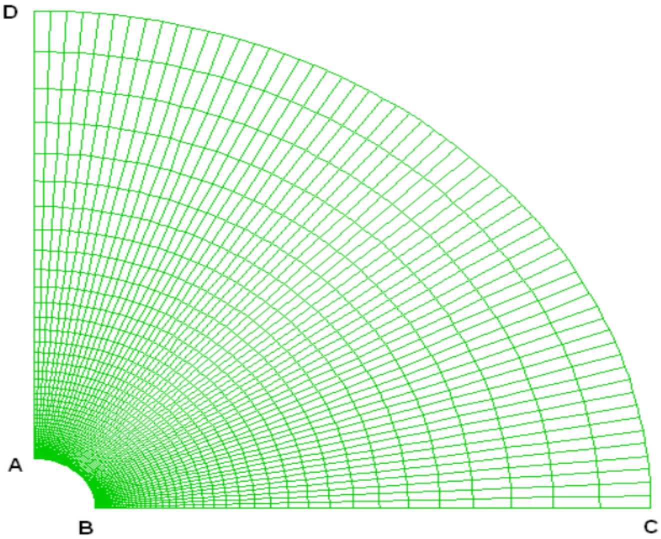

Figure 1. FLAC model for circular tunnel.

The FLAC model is shown in Figure 1. Considering the large pressure change near the inner wall, the network density is correspondingly increased, and is divided into 60 parts in radial direction and 60 parts in circumferential direction according to the equal ratio relation, totaling 3600 units. BC side has no longitudinal displacement, AD side has no lateral displacement, CD side applies 20MPa compressive stress. The initial stress of each element in the model is the hydrostatic pressure of 20MPa. The stress on the edge takes different values with different states. Due to the extremely symmetrical nature, the mechanical response of the whole tunnel can be analyzed with the calculation results on the sides of BC and BC.

If the Figure 1 and the model are changed from cylinder to solid cylinder, the model represents the surrounding rock and tunnel before excavation. Adopt Moore-Cullen Criteria and ideal elasto-plastic model are used to carry out elasto-plastic calculation for 1/4 and solid cylinder model separately. The results show that the original rock before excavation is in a uniform stress field, and in-situ stress will not make the original rock in a plastic state. Therefore, for the models of Figure 1 and Figure 1, the same 20MPa and pressure as in-situ stress can be applied to AB first to represent the force of rock in the space where the tunnel is located on the surrounding rock before excavation. The calculation process is divided into the following three steps:

In the first step, elastic-plastic calculation is carried out on the model before excavation. In this step, a uniform stress field is obtained, which is in agreement with the expectation. The results are consistent. The second step is excavation simulation. Keep other conditions unchanged, reduce the stress on AB to 0 and continue elastic-plastic calculation. This step simulates the process of tunnel excavation, because excavation means that the stress on AB is relieved. The third step is support simulation. Keep other conditions unchanged, apply 7MPa support stress on AB side, and continue elastoplastic. Sex calculation. In addition, the calculation of an elastic scheme is carried out separately: in Figure 1 and Model 1, the internal pressure of the edges of AB and AB is set to 7MPa, DC and edge External pressure is 0 and elastic calculation is made.

3.2 Calculation results and analysis

The calculation results of the second step and the third step respectively represent the mechanical state of the surrounding rock of the tunnel before and after support. From the calculated data extract BC and analyze the stress and displacement results on the edge. Obviously, BC and the horizontal stress of the tiny unit on the side is the radial principal stress of the tunnel, and the longitudinal stress is the circumferential principal stress. The stress distribution before and after support is shown in Figure 2.

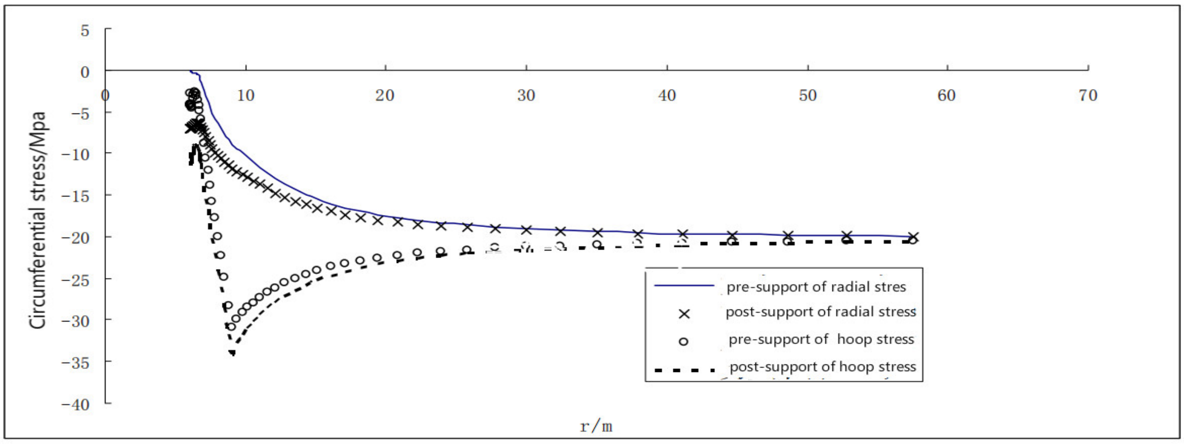

Figure 2. Comparison of radial and circumferential stresses versus radius for pre- and post- support.

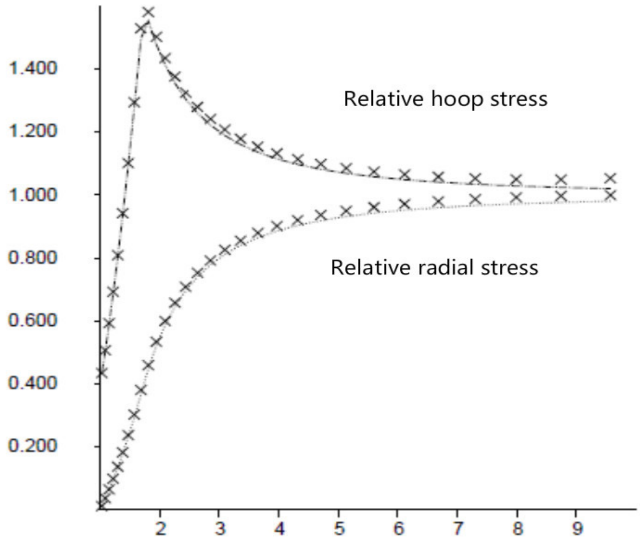

Figure 3 is an example of circular tunnel excavation in FLAC manual[7]. As can be seen from Figures 2 and 3, before support (i.e. excavation).

The distribution law of radial stress and circumferential stress in rear is similar to the results of numerical examples in FLAC and theoretical handbook. The surrounding rock after excavation has an elastic zone and a plastic zone. From Figure 2 it can be found that there is an obvious turning point on the circumferential stress curve. The stress curve on the right side of this point is similar to the stress curve of the tunnel under the elastic state. It can be seen that this point is the boundary between the plastic zone and the elastic zone. The radius at this point is 8.93 m, and the theoretical solution of plastic zone radius obtained from (2) formula is 8.85 m, which are basically the same, indicating that Karnofsky formula is applicable to tunnel mechanical analysis after excavation.

From the figure 2 and it can also be seen that the stress distribution law of surrounding rock of tunnel is similar before and after support, except that the absolute value of hoop stress decreases while the absolute value of radial stress increases. In other words, the difference between the circumferential stress and the radial stress is decreasing after the support, but the average value of the two is basically unchanged. This means that the radius of the stress circle at each point becomes smaller after support, but the position of the center of the circle remains unchanged. Obviously, the stress circle at this time is not tangent to the curves of the surrounding rock σ-τ described by Mohr Coulomb criterion, and the surrounding rock stress is not in a plastic state after support.

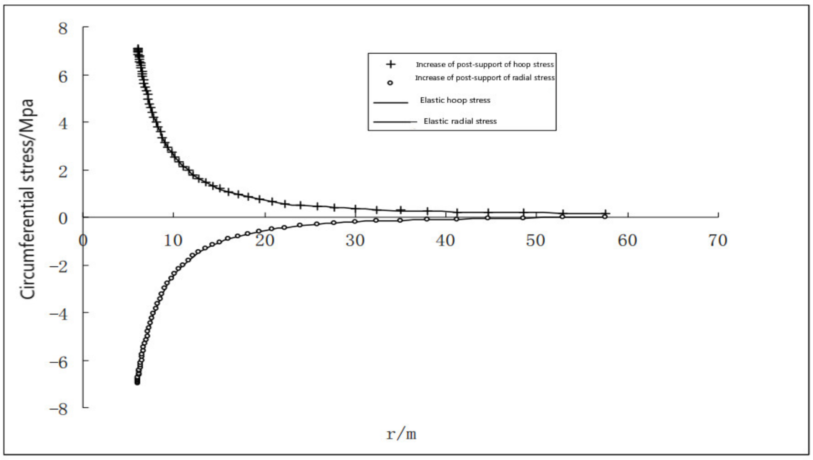

Figure 4. Support induced incremental stresses by plastic analysis and total stress by elastic analysis. (Note: in the elastic analysis, the model is subject to support stress only. Pressure on CD-the outer of FLAC model is zero).

In order to explain the stress change rule before and after support, the distribution curves of radial stress and circumferential stress increment along BC and BC after support are drawn in Figure 4 and the distribution curves of radial stress and circumferential stress along BC and BC under elastic conditions with only internal pressure but no external pressure are also drawn in Figure 4. It can be seen that the stress increment of tunnel surrounding rock after support is equal to the elastic stress caused by support stress. In other words, the stress distribution of surrounding rock after support is the stress distribution of elastic zone and plastic zone minus the elastic stress distribution before support.

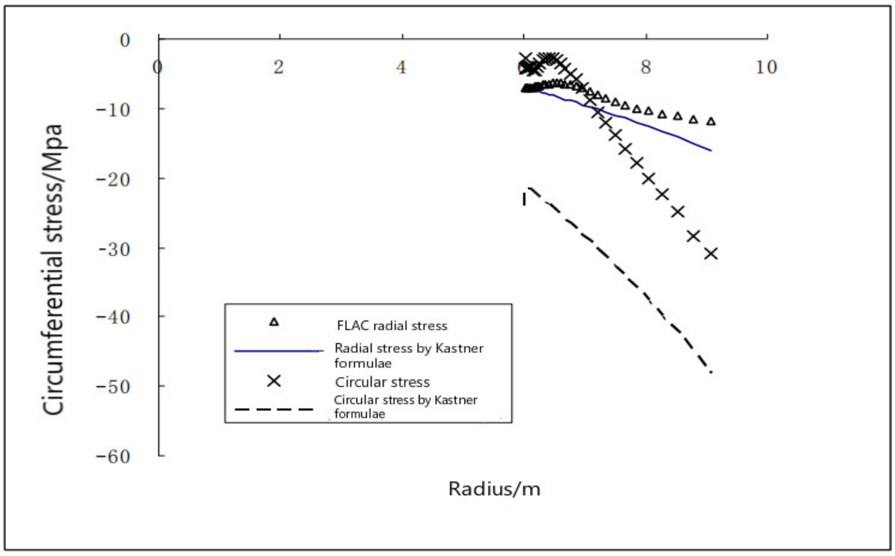

Figure 5. Residual stress and Stresses by Kastner formuale in plastic zone for post-support.

According to the definition of plastic mechanics, applying support belongs to unloading behavior, and the stress of surrounding rock after support is residual stress. For the original Figure 5 shows the residual stress calculated by FLAC and the stress calculated by Karnofsky formula, which are obviously different. The absolute values of radial and circumferential stresses calculated by Karnofsky"s formula are obviously greater than the corresponding values calculated by FLAC, which shows that Karnofskys formula is not applicable to stress calculation after support, and it can only describe the stress distribution law of surrounding rock after excavation.

For the judgment results of loading and unloading during excavation and support, we can understand it as follows: because the external pressure of the tunnel is always greater than that of the tunnel Internal pressure, so we should focus on external pressure or the difference between external pressure and internal pressure. The process of pressure difference increase is the loading process, and vice versa Unloading process. Excavation is the process of removing the initial internal pressure of the tunnel, which increases the pressure difference and produces the loading effect, so there is a plastic zone. However, the pressure difference in the supporting process will be reduced, thus unloading the surrounding rock, thus changing the plastic zone into a residual deformation zone.

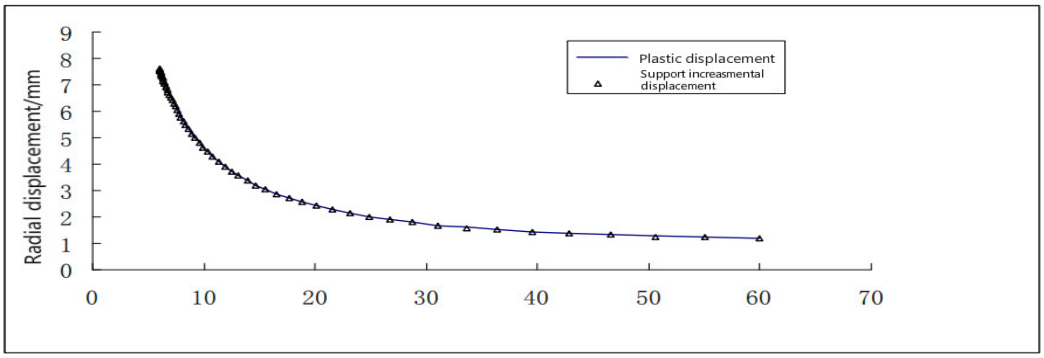

Figure 6. Support induced incremental displacement by plastic analysis and total displacemenr by elastic analysis. (Note: in the elastic analysis, the model is subject to support stress only. Pressure on the boundary CD in FLAC model is zero).

In terms of deformation, the displacement increment of surrounding rock caused by support and the distribution curves of displacement along BC and 6 in the elastic calculation scheme are all drawn in the figures. It can also be seen that the displacement of surrounding rock decreases after support, and the decrease conforms to the laws of elastic mechanics.

The above results show that the supporting process is the unloading process defined by plastic mechanics. This process will not produce new plastic zones. The stress in the original plastic zone will no longer meet the yield condition.

4. Conclusion

Through theoretical analysis and numerical calculation, the following new knowledge about tunnel construction can be obtained:

(1) The excavation process of tunnel is not unloading process, but loading process.

(2) Tunnel support is unloading process, which will not produce plastic zone, but will make the plastic zone formed by excavation become residual deformation zone.

(3) Karnofsky formula is only applicable to describe the stress distribution of surrounding rock after excavation, not after support.

The above conclusions will definitely have a significant impact on the tunnel design theory that has been followed for more than 60 years.

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