Government Policy, Factor Market Distortion and Structural Transformation

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Abstract: Demand-driven economic structural transformation is mainly realized through the Engel Effect, and different consumption has different income elasticity. This article attempts to explain the effects of taxation, technological progress and factor price distortions on economic structure by introducing government policies and capital labor price distortions into the multi-sectorial model. The results showed that the share of agricultural labor decrease when the tax rate decreased or technological progress occurred and the share of service labor increased when the non-homothetic of utility function was stronger. Similarly, the distortion of capital and labor factor prices will also affect the structural transformation, and the relationship between the two is opposite. When the distortion of manufacturing sector factor prices increases, the structural transformation will be accelerated. However, the structural transformation slows down as the distortion of factor prices in service industry increases.

Keywords: Tax Rate; Capital and Labor Price Distortion; Structure Transformation

1. Introduction

Structural transformation is often thought to be driven by two forces: income effects consistent with Engel-law (see Kongsamut, Rebelo, and Xie 2001; Foellmi and Zweimuller 2008) and relative price changes due to differences in productivity growth across sectors (see Baumol 1967; 2007; Acemoglu and Guerrieri, 2008)[1]. Dekle and Vandenbroucke found that government intervention also affected structural transformation, although their model did not distinguish between manufacturing and services. They suggest that, in a stable state, a decline in tax rates or a rise in sector productivity can lead to a decline in the share of agricultural employment. Whether these changes will lead to an increase in the share of employment in the service sector is an important question for developing economies.

In addition, the reality of factor markets is always distorted. In a perfectly competitive market, the price of an element is equal to the marginal product value of the element, while in reality the market is incomplete. In equilibrium, there are differences in the prices of elements in different sectors, which is the distortion of the element. Element distortion degree is not invariable. With the development of economy, the implementation of government policies, such as China’s household registration system reform and the increase of small micro enterprise credit, make element distortion degree change with the development of economy, the relative distortion of manufacturing and service sectors to change how to influence, the agricultural sector labor share, in turn, affects the industrial structure transformation.

In order to answer the above questions, this article constructs a three-sector model with distorted prices of government and capital labor factors. The research finds that the reduction of tax rate and technological progress will make the share of agricultural labor decrease faster and accelerate the structural transformation. In addition, when the non-

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homothetic of the utility function becomes stronger, the factor distortion degree of the manufacturing sector and the labor share of the service sector increase, and the balanced agricultural labor share decreases, accelerating the structural transformation. The variable effect of the factor distortion of the service sector is just the opposite.

2. Model

2.1 Household

We assume that individual in the economy is homogeneous. Consumers under a given budget constraint ensure the individual consumer to maintain life by consuming products to meet the agricultural products, manufactured goods and services. The demand of agricultural products is the lowest. In the service sector, family service, which is not involved in the market transaction, thus adding to the utility, has certain non-market services that Xie et al in the definition of the utility function. Therefore, the utility function of representative families is as follows.

\[ U = \sum_{t=0}^{\infty} \beta^t \left[ \alpha_a \ln(c_{at} - \tilde{a}) + \alpha_m \ln(c_{mt}) + \alpha_s \ln(c_{st} + \tilde{s}) \right] \quad (1) \]

Where \( w_a + w_m + w_s = 1 \), \( c_{at}, c_{mt}, c_{st} \) represents the consumption of agricultural products, manufactured goods and services respectively \( \beta \) is the time discount rate, which reflects the importance of future consumption; \( w_a, w_m, w_s \) represents the importance of consumption in the utility function; \( \tilde{a}, \tilde{s} \) represent the minimum consumption of agricultural products and the production of household services, respectively.

Constraints faced by consumers:

\[ p_{at}c_{at} + c_{mt} + p_{st}c_{st} + a_{r+1} = (1-r_t)[w_a h_{at} + w_m h_{mt} + w_s h_{st}] + (1+r_{at})a_{at} + (1+r_{mt})(1+\theta_{mt})a_{mt} + (1+r_{st})(1+\theta_{st})a_{st} \quad (2) \]

If the price of manufactured goods is normalized to 1, then \( p_{at}, p_{st} \) is the relative price of agricultural products and services; \( w_{at}, w_{mt}, w_{st} \) represents the price of labor in the three sectors, that is the actual wage rate; \( r_{at}, r_{mt}, r_{st} \) represents the prices of capital in the three sectors; \( h_{at}, h_{mt}, h_{st} \) represent the labor supply of the three sectors that the individual is willing to provide in the market, and assuming that the total labor supply is inelastic that is standardized as \( 1 \); \( a_{at}, a_{mt}, a_{st} \) represent an individual’s willingness to invest in three sectors; let’s assume that the capital and labor factor markets are distorted; using \( \rho_{mt}, \rho_{st} \) to characterize the relative distortions of the labor market in the manufacturing and service sectors and using \( \theta_{mt}, \theta_{st} \) to characterize the relative distortions of the capital market in the manufacturing and service sectors. The government regulates the economy through taxation, in which the tax rate is \( \tau_t, T_t \), is transfer payments for the government.

Willing to supply labor time constraints:

\[ h_{at} + h_{mt} + h_{st} = 1 \quad (3) \]

Under the given product price and resource constraint conditions, consumer utility maximization results:

\[ c_{at} : \quad \frac{\beta^t \omega_a}{c_{at} - \tilde{a}} + \lambda_t p_{at} = 0 \quad (4) \]

\[ c_{mt} : \quad \frac{\beta^t \omega_m}{c_{mt}} + \lambda_t = 0 \quad (5) \]

\[ c_{st} : \quad \frac{\beta^t \omega_s}{c_{st} + \tilde{s}} + \lambda_t p_{st} = 0 \quad (6) \]

\[ a_{r+1} : \quad \lambda_t = \lambda_{r+1}(1+\tau_{r+1})(1+r_{at+1}) \quad (7) \]

\[ h_{at} : \quad w_m(1-\rho_{ma}) = w_{at} \quad (8) \]

\[ h_{st} : \quad w_s(1-\rho_{st}) = w_{st} \quad (9) \]
\[ a_{mt} : r_{mt} (1 + \theta_{mt}) = r_{at} \]  
\[ a_{st} : r_{st} (1 + \theta_{st}) = r_{st} \]  

The euler-equation is obtained by combining (5) and (7)

\[ \frac{c_{mt+1}}{c_{mt}} = \beta (1 + \tau_{at}) (1 + r_{at}) \]

By combining the equations (4) and (5) and (5) and (6), we can obtain:

\[ c_{mt} = p_{at} c_{at} \left( \frac{\omega_{at}}{\omega_{a}} \right) \left( 1 - \frac{\bar{a}}{c_{at}} \right) \]
\[ c_{mt} = p_{st} c_{st} \left( \frac{\omega_{st}}{\omega_{s}} \right) \left( 1 - \frac{\bar{s}}{c_{st}} \right) \]

2.2 Producer

Producers produce under certain technical conditions by investing capital and labor, and the three departments all produce in the form of Cobb-Douglas:

\[ Y_{at} = A_{at} K_{at}^{\alpha} L_{at}^{\beta} \]  
\[ Y_{mt} = A_{mt} K_{mt}^{\alpha} L_{mt}^{\beta} \]  
\[ Y_{st} = A_{st} K_{st}^{\alpha} L_{st}^{\beta} \]

\[ Y_{at}, Y_{mt}, Y_{st} \] represents the output of the three sectors, \( A_{at}, A_{mt}, A_{st} \) represents the production technology of the three sectors, and \( \mu, \alpha, \gamma \) represents the output share of capital in agricultural, service and manufacturing production.[3]

At a given factor price, the producer maximizes profits by selecting capital and labor. The maximization results are as follows:

\[ K_{at} : \quad \mu p_{at} A_{at} K_{at}^{\beta} L_{at}^{\mu} = r_{at} + \delta \]  
\[ L_{at} : \quad (1 - \mu) p_{at} A_{at} K_{at}^{\beta} L_{at}^{\mu} = w_{at} \]  
\[ K_{mt} : \quad \alpha A_{mt} K_{mt}^{\beta} L_{mt}^{\mu} = r_{mt} + \delta \]  
\[ L_{mt} : \quad (1 - \alpha) A_{mt} K_{mt}^{\beta} L_{mt}^{\mu} = w_{mt} \]  
\[ K_{st} : \quad \gamma p_{st} A_{st} K_{st}^{\beta} L_{st}^{\mu} = r_{st} + \delta \]  
\[ L_{st} : \quad (1 - \gamma) p_{st} A_{st} K_{st}^{\beta} L_{st}^{\mu} = w_{st} \]

Combined equations (16) - (19) and (20) - (23) can obtain:

\[ \frac{K_{at}}{L_{at}} = \frac{L_{at}}{K_{at}} \frac{\mu}{1 - \mu} A_{at} \left( 1 - \rho_{at} \right) \frac{r_{at} + \delta}{\rho_{at}} \]  
\[ \frac{K_{mt}}{L_{mt}} = \frac{L_{mt}}{K_{mt}} \frac{\alpha}{1 - \alpha} A_{mt} \left( 1 - \rho_{mt} \right) \frac{r_{mt} + \delta}{\rho_{mt}} \]  
\[ \frac{K_{st}}{L_{st}} = \frac{L_{st}}{K_{st}} \frac{\gamma}{1 - \gamma} A_{st} \left( 1 - \rho_{st} \right) \frac{r_{st} + \delta}{\rho_{st}} \]

2.3 Government

The government needs to balance the budget every period, so there is

\[ T_{i} = \tau_{i} [w_{at} h_{at} + w_{mt} (1 - \rho_{mt}) h_{mt} + w_{st} (1 - \rho_{st}) h_{st} + T_{i} + (1 + r_{at}) a_{at} + (1 + r_{mt} (1 + \theta_{mt})) a_{mt} + (1 + r_{st} (1 + \theta_{st})) a_{st}] \]

Adjusting equation (26)

\[ T_{i} = \frac{\tau_{i}}{1 - \tau_{i}} [w_{at} h_{at} + w_{mt} (1 - \rho_{mt}) h_{mt} + w_{st} (1 - \rho_{st}) h_{st} + (1 + r_{at}) a_{at} + (1 + r_{mt} (1 + \theta_{mt})) a_{mt} + (1 + r_{st} (1 + \theta_{st})) a_{st}] \]
2.4 Equilibrium

In the government to regulate the economy through tax, capital and labor elements exist distortion on the market, namely the price of capital and labor factor is not equal to marginal cost, this will influence the individual to the supply of labor and capital, thus affecting economic growth need to satisfy the three conditions, an equilibrium is a given commodity prices, consumers through the choice to maximize its own utility consumption and investment, the second is for the enterprise, under the given factor prices, choice of labor and capital for production, so as to maximize profits, three also need to satisfy the product market clearing, namely

\[ Y = c \]  
(28)

\[ Y = c \]  
(29)

\[ h = L \]  
(30)

\[ h(1-\rho) = L \]  
(31)

\[ h = L \]  
(32)

\[ r = K \]  
(33)

\[ a(1+\theta) = K \]  
(34)

\[ a(1+\theta) = K \]  
(35)

\[ a = K + K / (1+\theta) + K / (1+\theta) \]  
(36)

\[ c = K + K / (1+\theta) + K / (1+\theta) \]  
(37)

The proof of equation (37) is given in appendix A.

We use charts to analyze the system steady-state, and analyze the changes in the labor share of the department when the tax rate, production technology and the degree of distortion of factor market change. According to the above analysis of equilibrium, the labor share of the agricultural sector at equilibrium can be determined by formula (38) below (certified in annexes B and C).

\[ \frac{r + \delta}{\delta} \frac{(1-\alpha)}{(1-\mu)} \alpha = (1-\frac{\alpha}{Y})(1-\frac{\alpha}{r + \delta}) - \frac{\alpha}{r + \delta} \frac{(1+\eta)}{(1+\theta)} \frac{\alpha}{Y} - \frac{\alpha}{1-\mu} \frac{(1-\alpha)}{(1-\mu)} \frac{\alpha}{r + \delta} \frac{(1-\alpha)}{(1-\mu)} \frac{\alpha}{r + \delta} \frac{(1-\alpha)}{(1-\mu)} \frac{\alpha}{L} \]

where

\[ r + \delta = \frac{1}{\beta(1-\tau)} \]
(39)

\[ r + \delta = \frac{1}{\beta(1-\tau)} \]
(40)

\[ L = \frac{1}{\rho} \]
(41)

\[ Y = A \alpha A \alpha \]
(42)

\[ Y = A \alpha A \alpha \]
(43)

According to equation (38), the left side (LHS) of the equation increases with the increase of agricultural labor share, while the right side (RHS) decreases with the increase of agricultural labor share, so there is a stable point in the...
economy. Let’s use the positive slope line from the origin to represent RHS, and the negative slope line to represent LHS, so the point where the two lines intersect is the agricultural labor share at a stable point. We first analyze the case \( \bar{a} \neq 0 \) and \( \bar{s} \neq 0 \) where the utility function is non-homothetic.

### 2.4.1 When \( \bar{a} \neq 0 \) and \( \bar{s} \neq 0 \)

Figure 1 shows that when the tax rate is lowered, the share of labor in the balanced agricultural sector declines. According to equation (38), the tax rate affects the equilibrium labor share through two ways. The other is non-homothetic of utility function, which is realized by \( \frac{\bar{a}}{\bar{s}} \neq 0 \); another one is that it appears directly in the equation, which affects the labor share at equilibrium through the equilibrium interest rate. Because the right-hand side of the equation goes up as the tax rate goes up, and the left-hand side goes down as the tax rate goes up, the share of labor in the agricultural sector goes down at equilibrium\[^4\]. However, the influence of labor share in service industry is uncertain, because in equilibrium \( L_s = (1 - \rho_s)\eta L_a \), tax rate decreases, agricultural labor share decreases, and \( \eta \) increases with the reduction of tax rate. This non-homothetic of utility function makes the labor share in service industry increase, so the change of labor share in service industry in equilibrium depends on which force is stronger. The influence of tax rate on labor share can be explained from the income effect, with the decrease of the tax rate, income will increase, non-homothetic utility function reflect that the income elasticity of consumption in sector of agriculture, manufacturing and services is less than or equal to or greater than 1, therefore, with the increase of income, agriculture, manufacturing and services consumption share will reduce, the same, and in turn increase, and then the labor share of agricultural sector may fall, and in the service sector it is likely to increase.

Figure 2 and Figure 3 analyze the impact of technological progress on the balanced labor share. Through (38) we can see technology progress only through \( Y_a, Y_e \) to influence the share of labor, the technology progress in agriculture and manufacturing department makes LHS and RHS line in figure 2 more steeper, then in the equilibrium, labor share of agriculture sector would decline, but in the service sector the change of labor share is uncertain, advances in technology have increased \( \eta \), and the agricultural labor share decrease, when the effect of technological progress is stronger, service industry labor share will increase, otherwise it will decrease; when service technology progress, as shown in Figure 3\[^5\], LHS line keep constant and RHS line more steep, and in equilibrium, the agricultural sector labor share decrease, similarly the effect on labor share of service industry is uncertain, when the power of labor shares’ falling of the agricultural sector is stronger than the increase of \( \eta \) caused by technological progress, consumption share in the service sector will decline\[^6\].

Figure 4 and Figure 5 analyze the effect of decline of labor market distortion on labor share in the equilibrium. According to formula (38), it can be seen that the labor market only influences the labor share by \( Y_a, Y_e \). When labor market distortions in the manufacturing sector decrease, both the LHS curve and the RHS curve will slow down, so that the labor share in the agricultural sector will increase at equilibrium\[^7\]. When the labor market distortions in the service sector decrease, the effect on the labor share in the equilibrium is the same as the technological progress in the service sector, that is, the LHS does not change, the RHS becomes steep, and in equilibrium, the agricultural sector labor share declines in the equilibrium. However, when the manufacturing labor market is less distorted, the change in the employment share of the service industry depends on the balance between the change in employment share of the agricultural sector and the non-homothetic of utility function\[^8\]. When the increase in the agricultural labor share is stronger, the employment in the service industry increases; otherwise, it decreases. When the distortion of the labor market in the service industry decreases, \( \eta \) increases, so the employment proportion in the service industry is also uncertain. When the \( \eta \) increasing force is stronger, the employment proportion in the service industry increases; in addition, the effect of labor market distortions in manufacturing and services on the share of employment in the agricultural sector is opposite, and the former is more conducive to structural transformation.

Figure 6 and Figure 7 analyze the effect of capital market distortion on the labor share in equilibrium. When the capital market distortion in the manufacturing sector increases, the agricultural labor share in equilibrium can be affected directly or through \( Y_a, Y_e \). The right side of equation (38) will decrease with the increase of this distortion, so the RHS curve will decline. The effect of this increase in distortion on LHS is uncertain. When the impact through \( Y_a \) is
large enough, the slope of LHS increases, and the share of agricultural labor declines at the equilibrium point, but \( \eta \) increases. Therefore, the employment proportion of service industry becomes uncertain, and the employment proportion of service industry increases when the force of \( \eta \) is greater. When the direct effect of factor distortion on LHS is large, and the slope of LHS decreases, and the change of labor share in the agricultural sector is uncertain\(^9\). When the distortion of the capital market in the service sector increases, the LHS remains unchanged and the RHS becomes slower. The share of agricultural labor increases, but \( \eta \) decreases. When the strength of \( \eta \) is strong, the share of employment in the service sector declines and the structural transformation slows down.

2.4.2 When \( \bar{a} = 0, \text{ and } \bar{s} = 0 \)

The preference is non-homothetic. A lower tax rate would reduce the equal share of labor in the agricultural and service sectors, while technological progress would not affect the equal share of labor. In reality, minimum agricultural products and consumption are needed in order to survive and co-travel. Meanwhile, the existence of household services is also inevitable. Moreover, due to the different nature of commodities, the consumption of these commodities cannot be increased in equal proportion when income increases.
3. Conclusion

This article constructs a three-sector model to analyze the effects of tax rate, technological progress and factor market distortion on economic structure. It was found that the labor share of the agricultural sector decreased when the tax rate decline and there was technological progress in the agricultural and manufacturing sectors, while the labor share of the service sector increases when the non-homothetic of utility function was strong enough. This conclusion has a strong policy effect. The government can stimulate consumption by lowering taxes or improve production efficiency by encouraging enterprises to develop and innovate, thus promoting structural transformation.[10]

Factor market distortion will also affect the employment share of the agricultural sector at equilibrium. The increase of factor distortion in the manufacturing sector can promote the structural transformation, while the increase of factor distortion in the service sector will slow the structural transformation.

References
Appendix

Prove equation (37)
substitute equation (27) into equation (2) to obtain
The above equation (30-36) can be expressed as follows
\[ p_a Y_a + p_m Y_m + K_{st} + K_{st+1} / (1 + \theta_{st+1}) + K_{st+1} / (1 + \theta_{st+1}) \]
\[ = w_{st} L_{st} + w_{st} L_{st} + w_{st} L_{st} + K_{st} + K_{st} / (1 + \theta_{st}) + K_{st} / (1 + \theta_{st}) + (a, K_{st} + r_s K_{st} + r_s K_{st}) \]  
(A.1)

Using equations (19), (21) and (23)
\[ w_{st} L_{st} + w_{st} L_{st} + w_{st} L_{st} = \left(1 - \mu \right) p_{at} Y_{at} + \left(1 - \alpha \right) Y_{at} + \left(1 - \gamma \right) p_{st} Y_{st} \]  
(A.2)

Using equations (18), (20) and (22)
\[ r_{st} K_{st} + r_{st} K_{st} + r_{st} K_{st} = \mu p_{at} Y_{at} + \alpha Y_{at} + \gamma p_{st} Y_{st} - \delta(K_{st} + K_{st} + K_{st}) \]  
(A.3)

Substitute equation (a.2) (a.3) into equation (a.1) to obtain equation (37)
\[ 1 + 1 + 1 \]
\[ / (1 + \theta_{at}) + / (1 + \theta_{at}) = / (1 + \theta_{at} + \theta_{st}) + / (1 + \theta_{st}) + \delta(K_{at} + K_{st} + K_{st}) \]

B. Prove (38) — (40)
The sum of capital can be converted into the following form
\[ K_a + K_m + K_s = \frac{K_m}{L_m} \left( \frac{K_m}{L_m} + L_m + \frac{K_m}{L_m} \right) \]
Equation (24) and (25) are substituted into the above equation
\[ K_a + K_m + K_s = \frac{K_m}{L_m} \left( \frac{1 - \alpha}{1 - \alpha} + (1 - \rho_m) \frac{r_m + \delta}{L_m} + \frac{\gamma}{1 - \gamma} \frac{1 - \alpha}{1 - \rho_m} \frac{r_m + \delta}{L_m} \right) \]  
(B.1)

The manufacturing production function can be written as follows
\[ Y_m = A_m \left( \frac{K_{st}}{L_m} \right)^\gamma L_m \]  
(B.2)

When the economy reaches steady state, it can be obtained from equations (12) and (27):
\[ c_m = Y_m - \delta(K_a + K_m + K_s) \]
Thus: economy reaches steady state
\[ r + \delta = \frac{1}{\beta(1 - \tau)} \]
\[ r + \delta = \frac{1}{\beta(1 - \tau)} - \frac{1}{1 + \theta_m} \]
\[ r + \delta = \frac{1}{\beta(1 - \tau)} - \frac{1}{1 + \theta_s} \]  
(B.3)

According to (B.1) (B.2), the steady state is
\[ c_m = A_m \left( \frac{K_m}{L_m} \right)^\gamma L_m - \delta K_m \left[ \frac{1 - \alpha}{1 - \alpha} + (1 - \rho_m) \frac{r_m + \delta}{L_m} + \frac{\gamma}{1 - \gamma} \frac{1 - \alpha}{1 - \rho_m} \frac{r_m + \delta}{L_m} \right] \]  
(B.4)

The combined equations (18) and (20) are
\[ p_a Y_a = \frac{\alpha}{\mu} \frac{K_m}{Y_m} \]  
(B.5)

The combined equations (24) and (b.2) can represent equation (b.5) as follows
\[ p_a Y_a = \frac{\alpha}{\mu} A_m \left( \frac{K_m}{L_m} \right)^\gamma \frac{1 - \alpha}{1 - \alpha} \frac{r_m + \delta}{L_m} \]  
(B.6)

Combined equations (13) and (28)
\[ c_m = \frac{\alpha_1}{\alpha_2} (1 - \frac{\pi}{\lambda}) p_a Y_a \]  
(B.7)

Combined equations (B.6) (B.7):
\[ c_m = \frac{\omega_a}{\omega_s} \left( 1 - \frac{\pi}{\gamma} \right) \alpha A_m \frac{K_m}{L_m} \left( 1 - \alpha \right) \frac{1}{1 - \alpha} \left( 1 - \rho_m \right) \frac{r_m + \delta}{r_m + \delta} L_m \]  

(B.8)

From equation (20)
\[ A_m \frac{\alpha}{r_m + \delta} = (\frac{K_m}{L_m})^{1-a} \]  

(B.9)

Combined equations (14) and (29)
\[ c_m = \frac{\omega_a}{\omega_s} \left( 1 + \frac{\pi}{\gamma} \right) p_a Y_s \]  

(B.10)

Combined equations (B.10) (B.7)
\[ \frac{p_a Y_s}{p_a Y_s} = \frac{\omega_s}{\omega_s} \left( 1 - \frac{\pi}{\gamma} \right) \]  

(B.11)

Combined equations (19) (23) (11)
\[ \frac{L_m}{L_m} = 1 - \gamma \omega_s \left( 1 - \mu \right) \frac{1}{Y_s} \left( 1 - \rho_s \right) = (1 - \rho_s) \frac{h_x}{h_a} \]  

(B.12)

Combined equations (B.9) (B.4) (B.8) and (3), (30)-(32), order \( \frac{h_x}{h_a} = \eta \) we can get:
\[ \frac{r_m + \delta}{r_m + \delta} (1 - \alpha) \frac{\omega_m}{\omega_s} (1 - \frac{\pi}{\gamma} Y_s) = (1 - \frac{\Delta}{r_m + \delta}) \left( 1 + \frac{\rho_s}{r_m + \delta} \right) \left( 1 + \frac{\rho_s}{r_m + \delta} \right) + \frac{\mu}{1 - \mu} (1 - \alpha) \frac{\delta}{r_m + \delta} + \eta(1 - \alpha) + \frac{\gamma}{1 - \gamma} \frac{\delta}{r_m + \delta} L_m \]

It is equations (38)

Prove (41) (42)

Adjust the production functions of agriculture and services as follows:
\[ Y_a = A_a \left( \frac{K_a}{L_a} \right)^{\gamma} L_a \]
\[ Y_s = A_s \left( \frac{K_s}{L_s} \right)^{\gamma} L_s \]  

(C.1)

Combine equations (24) (25) (C.1) (B.9) (B.3), adjusting equation (24) and (25). The capital-labor ratio in agriculture and service industry is expressed as the capital-labor ratio in manufacturing industry and substitute it into (C.1), by (B.9) making the capital-labor ratio of manufacturing is expressed in terms of technology and interest rates at last combing function (B.3). Substitute in the interest rate expression and we get the following output expression for agriculture and services:
\[ Y_a = A_a \left( \frac{\alpha}{\phi} \right)^{\mu} \left( \frac{1 - \alpha}{1 - \alpha} \right) \frac{1 - \rho_m}{1 - \rho_m} \left( \frac{r_m + \delta}{1 - \mu} \right)^{1 - \alpha} (1 - \beta) \frac{1 - \gamma}{1 - \gamma} \frac{1 - \rho_s}{1 - \rho_s} \left( \frac{r_m + \delta}{1 - \gamma} \right)^{1 - \gamma} L_m \]
\[ Y_s = A_s \left( \frac{\alpha}{\phi} \right)^{\mu} \left( \frac{1 - \alpha}{1 - \alpha} \right) \frac{1 - \rho_m}{1 - \rho_m} \left( \frac{r_m + \delta}{1 - \gamma} \right)^{1 - \gamma} \frac{1 - \rho_s}{1 - \rho_s} \left( \frac{r_m + \delta}{1 - \gamma} \right)^{1 - \gamma} L_s \]

They are functions (41) and (42).