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Mathematical Model of Akashi Kaikyo Bridges

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Abstract: This assignment examines the vital role of mathematical modelling in the design of a significant bridge structure. It explores the dynamic interplay between mathematics and architectural design. The report provides a thorough evaluation of the bridge, utilising both quadratic and an additional polynomial function to represent it. Throughout the development process, algebraic methods are employed, and advanced technology is integrated for precise results. Additionally, regression plots are utilised, enabling a comprehensive evaluation of the proposed models and ensuring they accurately reflect the bridge's architectural features. Ultimately, this report demonstrates the essential role of mathematical modelling in the design process, providing a detailed analysis of its use in the creation of a bridge structure.

Keywords: Mathematical Model;Mathematics Modeling;Mingshi Bridge

1. Introduction

Context The aim of this report is to investigate the Akashi Kaikyo Bridge, a suspension bridge that is of interest due to its length, as it's the world's longest suspension bridge. Specifically, the shape of the Akashi Kaikyo Bridge will be modelled using both quadratic and linear functions. The use of such mathematical models is a significant aspect of the bridge's engineering design and is worthy of further analysis and discussion. The purpose of this report is to provide a clear and concise overview of the Akashi Kaikyo Bridge, its design features, and the role of mathematical modelling in its construction and the mathematics in actual life.

2. Mathematical concepts and techniques

In order to solve this complex modelling problem, it is important to know relevant features of the graphs of linear, quadratic, polynomial functions, which is used to model the bridge, including their key points and the vary of functions due to the change of variable. There are 4 formulas used to model the arch of bridge.

2.1 Quadratic function

The standard formula of quadratic function is:

$$y = ax^2 + bx + c$$

The terms 'a', 'b', and 'c' are coefficients. where the coefficient a determines whether the parabola opens upwards or downwards and how wide it is. The coefficient 'b' affects the symmetry of the parabola and shifts it along the x-axis. The term 'c' is the constant term, which determines the vertical shift or the y-intercept of the parabola. 'x' is typically referred to as the independent variable. it is the input value used to determine the corresponding value of the dependent variable 'y'. by using the vertex to find the specific equation of quadratic function, vertex formula is also necessary.

The vertex formula of quadratic function is:

$$y = a(x-h)^2 + k$$

The value of h corresponds to the x-coordinate of the vertex, and the value of 'k' corresponds to the y-coordinate of the vertex.

2.2 Linear function

The standard formula of linear function is:

$$y = kx + b$$

'k' is the coefficient of the independent variable, x and represents the slope of the line. The slope determines how steep the line is. 'b' is the constant term and represents the y-intercept, which is the point where the line intersects the y-axis.

2.3 Polynomial function

In order to make the equation more consistent with the arch in the image, using one type of polynomial functions which the power is -0.1.

The general formula of polynomial function is:

$$y = ax^{-0.1} + b$$

'a' and 'b' are the coefficients. 'a' is the coefficient of the term $x^{-0.1}$, which represents the degree of the term. 'b' is the constant term, representing the y-intercept, which is the point where the function intersects the y-axis.

Additionally these functions will be applied in conducting the mathematical model and working out the results."Desmos" is to be used as a technology tool to construct the graphs of modelling the bridge.

3. SOLVE - RESULTS AND DISCUSSION

Creating quadratic function:

The quadratic function is used for modelling the arc of the middle suspension wire. Choosing two key points on the function is necessary for getting the specific function data. One of points must be the vertex if the vertex formula was used. The other point is allowed to choose randomly without demanding.

There are two ways to solve this equation set: Elimination and substitution. In this equation set, Elimination was used.

4. Evaluation

Evaluation of the reasonableness of solutions:

To examine the reasonableness of the model, the degree of fitness of the function models to the actual graph should be considered. To calculate the accuracy of the model, the percentage errors are used given that if the modelled line can fit the original arc, then every point on the arc should correspond to a point on the linear function. In order to calculate percentage error, an accurate point is plotted on the actual arch by eye-measuring. The x value of the accurate point is then subbed into the model function to obtain a y value. The y value is then compared to the y value of the accurate point by the following formula:

$$Percentage\ error = \frac{|y\ value\ given\ by\ the\ function - y\ value\ of\ the\ accurate\ point|}{|y\ value\ of\ the\ accurate\ point|} * 100\%$$

Plugging the value of x of -39.2 into the equation to get the y value given by the linear function:

$$y = \frac{166}{315} \times (-39.2) + \frac{12763}{630}$$

$$y = -\frac{152111351337}{381185963971}$$

So the percentage error of linear equation is:

$$P = \frac{|(-\frac{152111351337}{381185963971}) - 0.2|}{0.2} \times 100\%$$

$$P \approx 300\%$$

Repeat the steps for 2 times:

When x = -36, P $\approx$ 19.5%

When x = -24, P $\approx$ 57.1%

So the average of percentage error of linear equation is 125.6%.

Quadratic Function:

For the quadratic function, accurate points (15.5,10.5), (16,10.9), (16.9,11.7) are plotted. The percentage error are shown in the table below :

Repeat the steps for 3 times as same as the linear function:

When x = 16.9, P $\approx$ 2.3%

When x = 15.5, P $\approx$ 2.4%

When x = -36, P $\approx$ 2.2%

So the average of percentage error of quadratic equation is 2.3%.

Repeat the steps for 3 times as same as the linear function:

When x = 21.9, $P \approx 1.4\%$

When x = 20.5, $P \approx 1.4\%$

When x = -36, $P \approx 1.5\%$

So the average of percentage error of quadratic equation is 1.4%.

Comparing between the linear function, quadratic function and polynomial function. Both of the quadratic function and polynomial function has a tiny percentage error which are less than 3%. So the model of quadratic and polynomial functions are reasonable. Whereas, the linear function shows a huge percentage error with 300% which is 100 times the error of the other two functions and beyond the allowable margin of error. So the linear equation is unreasonable in this model. In addition, the reasonableness of the assumption is questionable as there is no way to verify whether the points plotted by eye-measurement is actually accurate. This potentially limits the validity of the model.

Strength： It is easier to construct the mathematical models, conduct the calculations and get the solutions because external factors like the eye-to-eye error and Incomplete symmetry are not taken into considerations.As mentioned in the assumptions, the data of the real bridge is proportional to the data of the bridge in Desmos and the symmetry of the bridge is exist.As a mathematical tool, Desmos shows the information and relationships between equations more intuitively. It helps to develop and model the function when only several points are given.

Limitation:The mathematical models developed in this report are not very accurate in reality because there exists the incomplete symmetry.The approximation of the parameters and relevant results during calculations could cause errors and has an impact on the accuracy of the solutions.The model of the arc can be different as only coordinates of relevant points are given. Therefore, the result could vary under different models and functions.

5. Conclusion:

This report provides the model of the arc of Akashi Kaikyo Bridge for three parts of suspension arcs, by developing mathematical models with four various formulas of the quadratic function, linear function and polynomial function. The formula In the report, Desmos is used as a technological tool to draw the graphs of relevant equations. Strengths and limitations of the mathematical models and design are also discussed as well as the evaluation of accuracy of the bridge model. Overall, there are still more opportunities for the models and the design to develop and improve to be more accurate and suitable.

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