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Comparison of Several Calculation Formulas of Circular Arch Area

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Abstract: In our daily life and work, we often meet the closed figure of arc and straight line. The measurement and calculation of arcuate area is unavoidable. In this paper, the theoretical formula based on the center angle and radius of the circle, the calculation formula based on the arc length and chord length, the accurate calculation formula and approximate calculation formula based on the arc length and height, three calculation formulas based on the chord length and height, and the approximate calculation formula based on the arc length, chord length and height are given. The calculation formulas are compared for the convenience of practical application.

Keywords: Mathematics; Arcuate Area; Circle

In our daily life, we often meet the closed figure of arc and straight line. How to measure and calculate the area of an arch is a problem in front of us. For example, the calculation of the volume of a horizontal cylindrical tank in the function part of the new high school mathematics (compulsory one) involves the calculation of the arch area. This paper introduces and compares various formulas for calculating the area of arch.

1. Representation based on center angle and radius of circle

The concept of arcuate first appeared in the middle school geometry about the content of the circle, which is related to the concept of fan. The arcuate area can be expressed as the difference between the sector area and the triangle area, namely

$$S_{\text{弓}ANB} = S_{\text{扇}OANB} - S_{\Delta OAB} \tag{1}$$

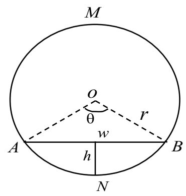

Figure 1. The arcuate area

With the help of the center angle θ and the radius r of the circle, it is easy to obtain

$$S_{\text{弓}ANB} = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta \tag{2}$$

Equation (2) the corresponding center angle θ is not more than π, and when $\pi \le \theta \le 2\pi$, there is a

$$\begin{aligned} S_{\text{弓}AMB} &= S_{\text{圆}} - S_{\text{扇}ANB} = \frac{2\pi}{2}r^2 - [\frac{1}{2}r^2\theta' - \frac{1}{2}r^2\sin\theta'] \\ &= \frac{2\pi-\theta'}{2}r^2 + \frac{1}{2}r^2\sin(2\pi-\theta') = \frac{1}{2}r^2\theta + \frac{1}{2}r^2\sin\theta \end{aligned} \tag{3}$$

Where $\theta' = 2\pi - \theta$, the formula (2) and (3) are combined to obtain

$$S_{\text{弓形}} = \frac{1}{2}r^2\theta + \frac{1}{2}\frac{\theta-\pi}{|\theta-\pi|}r^2\sin\theta \tag{4}$$

Equation (4) is the area representation of the arch based on the center angle and radius of the circle.

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2. Calculation based on arc length and chord length

In some practical cases, it is difficult to obtain the center angle of the arch, but it is easy to measure the arc length and chord length of the arch. In this case, the successive approximation method can be used to calculate the area of the arch.

Let the chord length and arc length of an arch be w and C respectively

$$\text{Because}\quad C = r\theta,\quad w = 2r\sin(\theta/2)$$

$$\text{so}\quad \theta/2 = C/w \cdot \sin(\theta/2) \tag{5}$$

In this case, if π is taken as the initial value of θ and substituted into equation (5), the estimated value of θ can be obtained as $2C/w$, and then the estimation of θ can be taken as the initial value to replace formula (5), that is to say

$$\theta_{n+1} = 2 \cdot C/w \cdot \sin(\theta_n/2) \tag{6a}$$

If used

$$\theta_{n+1} = 2 \cdot \arcsin(w\theta_n/2/C) \tag{6b}$$

The arcsin () function has no solution.

According to the reference, if the iteration is repeated for about 10 times, the difference between the estimated value of θ and the initial value will reach the allowable range of engineering approximate calculation. In fact, formula (6b) is not the only recursive solution of θ.

In consideration of

$$(r-h)/r = (C/\theta - h)/(C/\theta) = \cos(\theta/2) \tag{7}$$

It can also be determined θ by the following recursive formula, and its convergence rate is better than that of formula (6b).

$$\begin{cases} \theta_{n+1} = C \cdot (1-\cos(\theta_n/2))/h \\ \theta_0 = \pi \end{cases} \tag{8}$$

Finally, the arcuate area is calculated according to formula (9)

$$\begin{aligned} S_{弓形} &= \frac{\theta}{2}r^2 - \frac{w}{2}\sqrt{r^2 - \frac{w^2}{4}} \\ &= \frac{C^2}{2\theta} - \frac{w}{2}\sqrt{\frac{C^2}{\theta^2} - \frac{w^2}{4}} \end{aligned} \tag{9}$$

3. Approximate calculation formula based on chord length and height

In more practical cases, we measured the chord length and height of the arch, which can also be used to calculate the area of the arch. Among them, a commonly used approximate formula is as follows

$$S_{弓形} = \frac{2wh}{3} + \frac{h^3}{2w} \tag{10}$$

In reference, the approximate formula (10) was proved by using Zu Chongzhi's cyclotomy. The reference proves theoretically that formula (11) is closer to the true area of the arch than equation (10). It is verified by the author that the relative error of equation (11) can reach 1%.

$$S_{弓形} = \frac{2wh}{3} + \frac{8h^3}{15w} \tag{11}$$

The study in reference shows that the relative error of equation (12) is less than 0.3% when 0° is less than θ< 177° and is better than equation (10).

$$S_{弓型} = \frac{2}{3}wh[1 + 0.73(\frac{h}{w})^2] \tag{12}$$

4. Accurate calculation formula based on chord length and height

The concept of anti sine function is applied to equation (2)

$$S_{弓形} = \frac{1}{2} r^2 \theta - \frac{1}{2}(r-h)w$$
$$= 2r^2 \arcsin(\sqrt{\frac{h}{2r}}) - (r-h)\sqrt{h(2r-h)} \tag{13}$$

Equation (13) shows that when the radius of a circle is determined, the area of the arch is a function of its height. Equation (13) can be applied to the calculation of the area of an arch whose center angle is less than a semicircle, or when the central angle of a circle is greater than a semicircle. The storage capacity of horizontal cylindrical oil tank can be calculated very accurately by using equation (13).

According to Figure 1, it can be deduced that the following relationship exists between the radius of the circle and the height and chord length of the arch

$$r = \frac{h}{2} + \frac{w^2}{8h} \tag{14}$$

Substituting equation (14) into equation (13), there are

$$S_{弓型} = 2(\frac{h}{2} + \frac{w^2}{8h})^2 \arcsin(\sqrt{\frac{1}{1+(\frac{w}{2h})^2}}) - \frac{w}{2}(\frac{w^2}{8h} - \frac{h}{2}) \tag{15}$$

Compared with the approximate formula, it is obvious that the calculation of equation (15) is much more complicated. In addition, using the knowledge of calculus, we can deduce the homogeneous and accurate calculation formula of equation (15), which will not be discussed in this paper.

5. The approximate calculation formula of comprehensive arc length, chord length and height is shown in reference

$$S_{弓型} = 0.4613(whC)^{\frac{2}{3}} \tag{16}$$

According to reference, the relative error of formula (16) in calculating the area of arch is only 0.13%. However, formula (16) needs more measurement, and the error will be larger in actual use, so it has no obvious advantage compared with formula (12).

6. Conclusion

This paper summarizes the expression formula of arc area based on the center angle and radius of circle, the calculation formula based on arc length and chord length, approximate calculation formula and accurate calculation formula based on chord length and height. Among them, when the height is not easy to measure, the calculation formula based on arc length and chord length is applicable, see equation (9); after comparison, it is considered that equation (12) is better for approximate calculation based on arc length and height; for area expression based on arc length and height, accurate calculation formula is given, which is convenient for practical application.

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