Universe Scientific Publishing

ORIGINAL ARTICLE

Various Solutions and Visualization of a Limit Problem

Peiying Wang, Zhen Su, Zidong Lu, Jiaqi Liang, Jiachao Chen, Qinghe Xu, Yuan Wang

Guangzhou Institute of Technology, Guangzhou 510540, Guangdong, China.

Fund Project: school-level quality resource sharing project (2017 XJJG06); school-level teaching achievement award cultivation project (2019 JXJPY020).

Abstract: Limit is the basis of learning higher mathematics and one of the important contents of university learning higher mathematics. It is the examination content in graduate examination and the indispensable content in modern engineering work and scientific research. A variety of methods are used to solve the problem of a power function, and the correctness of the results is verified by MATLAB software.

Keywords: Limit; Equivalent Infinitesimal; Lopida Rule; Half-angle Formula; Taylor Formula

In order to help students learn higher mathematics, we set up an advanced mathematics learning group to ask students to ask and solve problems. Once, a classmate asked a question about the limit:

$$L=\lim_{x\to 0}\frac{\sqrt{1+x\sin x}-\cos x}{\sin^2(x/2)}$$

This is a limit problem with trigonometric functions and root functions. And when x tend to zero, $\sqrt{1+x\sin x}-\cos x$ and $\sin^2\frac{x}{2}$ the limit is 0, meet the condition of using equivalent infinitesimal quantity. Some students use relationships directly $\lim\limits_{x\to 0}\cos x=1$.

$$L=\lim_{x\to 0}\frac{\sqrt{1+x\sin x}-1}{(x/2)^2}=4\lim_{x\to 0}\frac{\frac{1}{2}x\sin x}{x^2}=2$$

This is a wrong answer. The reason is that the numerator and denominator are of the same order of magnitude and can’t be used alone $\lim\limits_{x\to 0}\cos x=1$.

In addition to using equivalent infinitesimal quantities, different formulas are used to form different methods. The following is a summary of the methodology.

1. Method 1: Use the square difference formula

$$L=\lim_{x\to 0}\frac{(\sqrt{1+x\sin x}-\cos x)\cdot(\sqrt{1+x\sin x}+\cos x)}{\sin^2(x/2)(\sqrt{1+x\sin x}+\cos x)}$$

$$=\lim_{x\to 0}\frac{1+x\sin x-\cos^2 x}{(x/2)^2(\sqrt{1+x\sin x}+\cos x)}$$

doi: 10.18686/ahe.v5i3.3475

$$= 4\lim_{x\to 0}\frac{\sin x(\sin x + x)}{x^2(\sqrt{1+x\sin x}+\cos x)}$$

$$= 4\lim_{x\to 0}\frac{\frac{\sin x}{x}\left(\frac{\sin x}{x}+1\right)}{\sqrt{1+x\sin x}+\cos x}$$

$$= \frac{4\times(1+1)}{\sqrt{1+0}+1} = 4$$

2. Method 2: 1 by 1

$$L = \lim_{x\to 0}\frac{\sqrt{1+x\sin x}-1}{\sin^2(x/2)} + \lim_{x\to 0}\frac{1-\cos x}{\sin^2(x/2)}$$

$$= \lim_{x\to 0}\frac{\frac{1}{2}x\sin x}{(x/2)^2} + \lim_{x\to 0}\frac{\frac{1}{2}x^2}{(x/2)^2}$$

$= 2 + 2 =4$

3. Method 3: Using the lopida rule

$$L = 4\lim_{x\to 0}\frac{\sqrt{1+x\sin x}-\cos x}{x^2}$$

$$= 4\lim_{x\to 0}\frac{\frac{\sin x + x\cos x}{2\sqrt{1+x\sin x}}+\sin x}{2x}$$

$$= 4\lim_{x\to 0}\frac{\sin x + x\cos x}{4x\sqrt{1+x\sin x}} + 4\lim_{x\to 0}\frac{\sin x}{2x}$$

$$= \lim_{x\to 0}\frac{\sin x + x\cos x}{x} + 2\lim_{x\to 0}\frac{\sin x}{x}$$

$$= \lim_{x\to 0}\frac{\sin x}{x} + \lim_{x\to 0}\cos x + 2\lim_{x\to 0}\frac{\sin x}{x}$$

$=1+1+2=4$

4. Method 4: Using the half-angle formula

$$L = \lim_{x\to 0}\frac{\sqrt{1+x\sin x}-\cos x}{1-\cos x}\times 2$$

$$= \lim_{x\to 0}\frac{(\sqrt{1+x\sin x}-1)+(1-\cos x)}{1-\cos x}\times 2$$

$$= \lim_{x\to 0}\left[\frac{(x\sin x)/2}{x^2/2}+1\right]\times 2 =4$$

5. Method 5: Using Taylor's formula

Using the Taylor formula: $\sqrt{1+x\sin x}=1+\frac{1}{2}x\sin x+o(x\sin x)$, $\cos x=1-\frac{1}{2}x^2+o(x^2)$, Using equivalent infinitesimal: $x\to 0$, $\sin^2\frac{x}{2}\sim\frac{1}{4}x^2$,

So the original

$$L=\lim_{x\to 0}\frac{1+(x\sin x)/2+o(x\sin x)-[1-x^2/2+o(x^2)]}{x^2/4}$$

$$=4\lim_{x\to 0}\frac{x(\sin x+x)/2+o(x\sin x)-o(x^2)}{x^2}$$

$$=2\lim_{x\to 0}\left[\frac{\sin x}{x}+1+\frac{o(x\sin x)}{x^2}-\frac{o(x^2)}{x^2}\right]=4$$

6. Validation of results-visualization

By using the MATLAB design program (as shown below), the limit can be calculated and the function curve can be drawn. As shown in Figure 1, the function is an even symmetric curve with a *x* trend to 0. The result calculated by MATLAB is exactly the same as that calculated by manual, which proves the correctness of manual calculation.

```
Simple drawing method and limit of         % Function
clear,syms x                               %Clear variables, define symbolic variables
f=(sqrt(1+x*sin(x))-cos(x))/(sin(x/2))^2;  %Sign function
L=limit(f,x,0)%Limit
figure, ezplot(L,[-1,1])                   %Create a graphics window, draw the limit level
hold on; ezplot(f,[-1,1])                  %Hold attributes, draw function curves
grid on                                    %Add grid
```

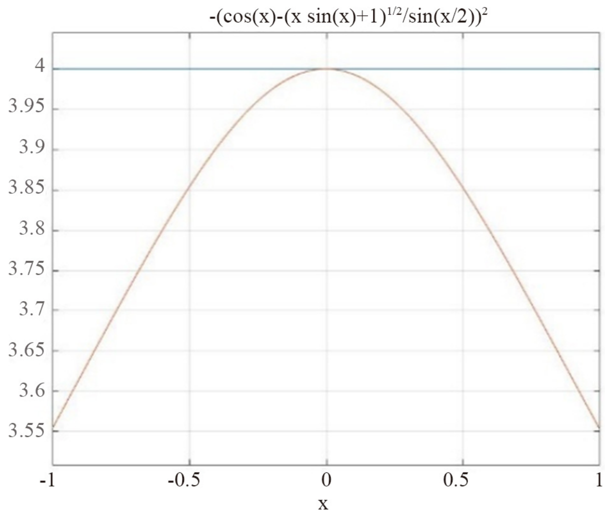

Figure 1. The function curve.

7. Conclusion

Use a variety of solutions to find the limit, open up students' thinking. Flexible use of square difference formula, equivalent infinitesimal. The quantity and half angle formula can find the limit skillfully, and the lopida rule and Taylor formula also provide a good idea in solving the limit problem. Learning and communicating in groups is a good learning method.

References

1. Zhou Q, *et al*. MATLAB visualization advanced mathematics (Vol.1). Hunan: Hunan University Press; 2011.
2. Department of mathematics Tongji University. Higher Mathematics. Version 7. Beijing: Higher Education Press; 2014.