

# Evolution of Cooperation Based on Complex Network

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**Abstract:** How does cooperative behavior evolve from selfish individuals? In order to explore the mechanism of cooperative behavior, we simulate the prisoner's dilemma, snowdrift game, kinselection, direct reciprocity and indirect reciprocity on complex networks.

**Keywords:** Complex network; Game; Evolution of cooperation

## 1. Introduction

Cooperation is very common in nature, from single celled microorganisms to advanced mammals, from social insects to human society, and the cooperation between biological individuals increases the survival opportunities of the whole population. However, according to Darwinism, natural selection is based on competition, and individuals will selfishly maximize their own interests, which obviously cannot explain the ubiquitous cooperative behavior. In order to understand how cooperative behavior evolves from selfish individuals, the exploration of the mechanism of cooperative behavior has aroused extensive attention and interest in academia. For example, Nowak et al. proposed five mechanisms of evolutionary game: kin selection, direct reciprocity, indirect reciprocity, network reciprocity and group selection.

Complex networks provide a convenient framework for describing game relations. Nodes represent game individuals and edges represent game relations. At each time step, the node plays a game with all its neighbors, accumulating the benefits of the game, and then updates the strategy according to the update rules, so as to repeat the iteration. We will study evolutionary game based on complex networks.

## 2. Model

(1) Establish a small world model: a nearest coupling network, which randomly reconnecting the key, which stipulates that there can only be one edge between any two different nodes, and each node cannot have an edge to connect with itself.

(2) Assign random value to each point:  $s(C)=1$ ,  $s(D)=-1$ , C for collaborator and D for betrayer.

(3) Game, calculate the profit of each point: the payoff matrix of game between two individuals  $\begin{matrix} f_1 & f_2 \\ f_3 & f_4 \end{matrix}$ .

(4) Strategy update and pairing comparison of each point: that is, the individual randomly selects a neighbor to compare the benefits, and changes the strategy from one probability to the other. Each node ( $P_1$ ) randomly selects a neighbor node ( $P_2$ ), and a certain probability  $w$  imitates its strategy. The common evolution rules are as follows:

$$W_{P_1 \leftarrow P_2} = \frac{1}{e^{\frac{P_1 - P_2}{k}} + 1}$$

Where  $P_i$  represents the cumulative income of  $i$ , parameter  $\kappa$  for noise, it represents the possibility of irrational behavior, which is usually a very small value, usually 0.1.

(5) Statistical results; including statistical diversity and average income of each round.

### 3. Results and analysis

(1) Prisoner's dilemma; the return matrix is:  $\begin{pmatrix} b^{-c} & -c \\ b & 0 \end{pmatrix}$ , let  $b = 3, c = 1$ , then we get Figure 1 by simulation.

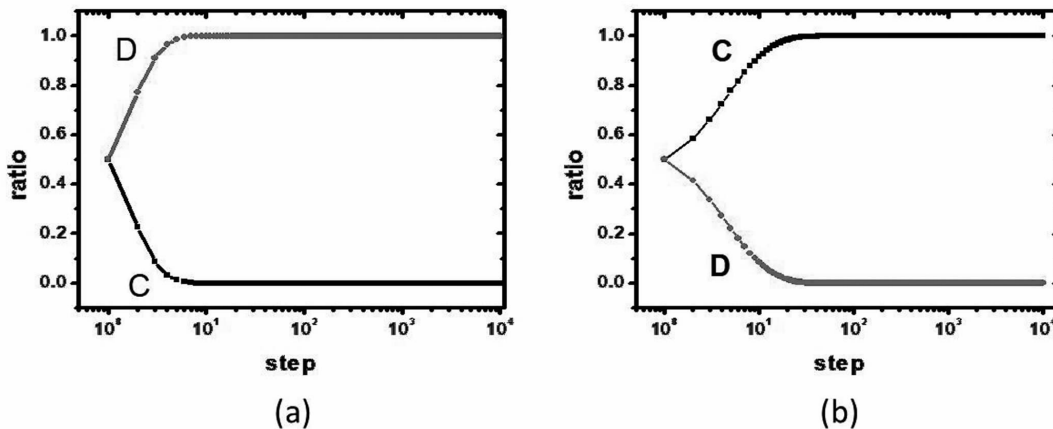


Figure 1. Relationship between population density and time in (a) prisoner's dilemma and (b) snowdrift game.

It can be seen from the figure 1(a) that the density (ratio) of C cooperators rapidly becomes zero, and the density (ratio) of D defectors rapidly becomes 1, which shows that in the prisoner's dilemma, cooperators die out rapidly, and fraudsters win the game, that is to say, the prisoner's dilemma cannot lead to cooperation. This result confirms the conclusion that the best strategy of prisoner's dilemma is betrayal. So where do so many examples of cooperation come from? Let's study its internal mechanism through several examples.

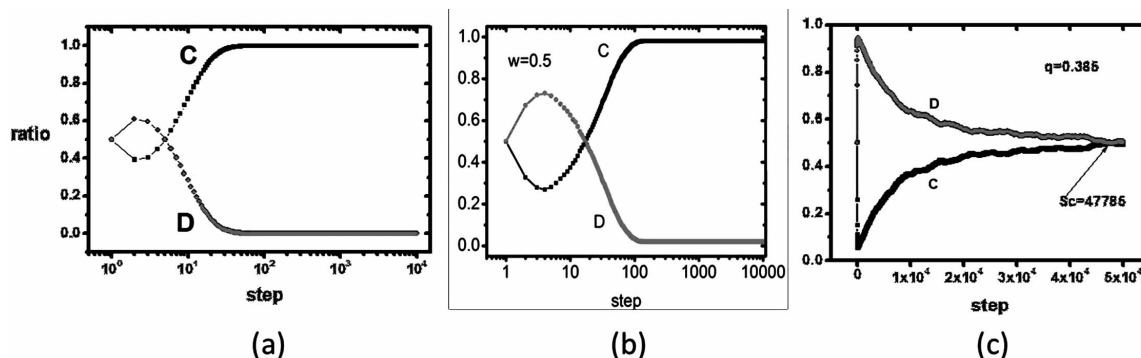
(2) Snowdrift game, its income matrix is:  $\begin{pmatrix} b - \frac{c}{2} & b - c \\ b & 0 \end{pmatrix}$ , let  $b = 3, c = 1$ , then we can get Figure 1(b) by program calculation.

It is obvious from the figure that the cooperators gradually occupy the group with the continuous progress of the game, which shows that the best strategy of snowdrift model in small world network is cooperation. The snowdrift model is different from the prisoner's dilemma: the profit of collaborator is higher than that of Betrayer. Therefore, a person's best strategy depends on the opponent's strategy: if the opponent chooses to cooperate, his best strategy is betrayal; Conversely, if the opponent chooses betrayal, then his best strategy is cooperation. In this way, cooperation will not die out in the system, and compared with the prisoner's dilemma, cooperation is easier to emerge in the snowdrift game.

(3) Kin selection; kin selection means that kinship can promote cooperation among individuals. The return matrix is:  $\begin{pmatrix} (b-c)(1+r) & br-c \\ b-rc & 0 \end{pmatrix}$ ,  $r$  is the probability of kinship, let  $r = 0.5, b = 3, c = 1$ , then we can get Figure 2(a) by program calculation.

It is obvious from the figure that the cooperators gradually occupy the group with the continuous progress of the game, which shows that kin selection can form cooperation. Of course, this conclusion has a certain premise, that is,  $r > c / b$ . Only when individuals are close enough, can kin selection work. In real life, such as father son relationship, brother relationship, friend relationship, stranger relationship, the intimacy of these relationships is getting smaller, we can see that their probability of cooperation is also getting smaller.

(4) Direct reciprocity; even if you work with me, I will choose to cooperate with you. The return matrix is;



**Figure 2.** Relationship between population density and time in (a) kinselection, (b) direct reciprocity and (c) indirect reciprocity.

$\begin{pmatrix} (b-c)(1-\omega) & -c \\ b & 0 \end{pmatrix}$ ,  $\omega$  is the probability of game again. If  $\omega=0.5$ ,  $b=3$ ,  $c=1$ , we can calculate the results by program as shown in Figure 2(b). The relationship between population density and time in direct reciprocity can be seen that the number of partners decreases first and then increases gradually until the whole population is occupied. Further study shows that the critical value of  $\omega$  is around  $c/b$ .

(5) Indirect reciprocity: indirect reciprocity refers to the good reputation formed by helping others, which is conducive to promoting the cooperation between individuals. The return matrix is:  $\begin{pmatrix} (b-c) & -c(1-q) \\ b(1-q) & 0 \end{pmatrix}$ ,  $q$  is reputation coefficient.

After calculation, we get the critical value of reputation coefficient is 0.385, when the reputation coefficient is lower than 0.385, betrayal will become dominant, and when it is higher than 0.385, cooperation will eventually become dominant. As shown in Figure 2(c), when  $q = 0.385$ , the population reaches equilibrium at the slowest speed, and the population density of cooperators and betrayers is the same (ratio = 0.5).

## 4. Conclusion

Based on the complex network, this paper mainly studies how to derive cooperation from the competitive system through five models: Prisoner's dilemma, snowdrift game, kinselection, direct reciprocity and indirect reciprocity. The best strategy of prisoner's dilemma is betrayal, and cooperation is easier to emerge in the snowdrift game. Only when individuals are close enough, kin selection can work and cooperation can be formed. In direct reciprocity, the number of cooperators first decreases, then gradually increases until they occupy the whole population, and the greater the chance of game again, the more conducive to cooperation. Whether the indirect reciprocity mechanism can support the cooperation depends on the reputation coefficient. Only when the reputation coefficient exceeds the critical value, the emergence of cooperation can be promoted.

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