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Design of a Programming Exercise Problem about Markov Chain

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***Abstract*:** In order to train students' programming practice ability and enhance their interest in learning, we design a classic Markov chain exercise as an online evaluation programming question.

***Keywords*:** Markov Chain; Automobile Insurance System; Programming Exercise

The concepts and theories related to Markov chain are relatively abstract. If you want to have good learning effect, you must practice, and almost all textbooks only provide exercise training, which is very boring. We design relevant exercises as programming questions and upload them to the online evaluation platform for students to carry out practical training in the learning process, which not only strengthens students' practical ability, but also improves their learning passion.

The following is a classic question about good-bad automobile insurance system. We construct data and design it as a programming question online evaluation platform.

1. Question background

The vast majority of annual car insurance benefits in Europe and Asia are determined by the so-called good-bad system. Each insured person is given a status of positive integer value, and the annual premium is a function of this status. The status of the insured changes year by year with the number of claims required by the insured. Because the low status corresponds to the low annual insurance premium, if the insured has no claim settlement requirements in the previous year, his status will be reduced, while if the insured has at least one claim settlement requirements in the previous year, his status will generally increase. Therefore, no claims are good and generally lead to low insurance benefits, while claims are bad and generally lead to higher insurance benefits.

2. Question description

For a given good-bad system, $S_i(k)$ record the status of an insured who is in status i in the previous year and has k claims in that year in the next year. The following table details a good-bad system assuming four states.

For example, this table describes $S_2(0) = 1$; $S_2(1) = 3$; $S_2(k) = 4, k \geqslant 2$.

For such a good-bad auto insurance system in four states, if the average number of claims per year is λ Poisson random variable, calculate the average annual premium paid by the insured.

		Next status			
status	**Annual insurance premium**	**0 claims**	**1 claim**	**2 claims**	**More than 3 claims**
1	200	1	2	3	4
2	250	1	3	4	4
3	400	2	4	4	4
4	600	3	4	4	4

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3. Input and output requirements and examples

Input format: one real number per line λ.

Output format: a real number P in one line indicates the average annual premium paid by the insured.

Input / output example: input 0.5; Output 326.375.

Description / tips:

[data range] $0 < \lambda < 2$

[special note] the answer with an error of no more than 10^{-2} is accepted. Please keep at least 3 decimal places for output.

[special nature] this question guarantees that the given data are solvable.

4. Solution method

The number of claims required by the insured per year is λ. The successive states of Poisson random variables form a Markov chain. The probability that the insured has k claims in a year is $a_k = e^{-\lambda}\frac{\lambda^k}{k!}$. The transition probability matrix of successive states of the insured is

$$P = \begin{bmatrix} a_0 & a_1 & a_2 & 1-a_0-a_1-a_2 \\ a_0 & 0 & a_1 & 1-a_0-a_1 \\ 0 & a_0 & 0 & 1-a_0 \\ 0 & 0 & a_0 & 1-a_0 \end{bmatrix}$$

Taking π_j as the proportion of the long-range time that the Markov chain stays in state j, then the equation satisfied is

$$\pi_j = \sum_i \pi_i P_{i,j}$$

$$\sum_j \pi_j = 1$$

The transition probability matrix can be obtained, and then the stationary probability can be obtained by Gaussian elimination method or matrix multiplication, so as to calculate the average annual premium paid by the insured. Suggested codes are as follows:

```
# include < bits/stdc+ + .h>
# include< math.h>
# include< stdio.h>
usingnamespacestd;
double x, zz;
double B[3];
intfactorial(int num)
{
if (num = =  0)
return1;
else
return num *  factorial(num - 1);
}
constint mod= 1e9+ 7;
int n;
double k;
structMat{
intr,c;
double A[105][105];
}ANS;
voidm_new(Mat &p,intr,int c){
p.r= r,p.c= c;
memset(p.A,0,sizeof(p.A));
}
Mat mul(Mat A,Mat B){
```

```
Mat C;
m_new(C,n,n);
for(int i= 1;i< = n;+ + i)
for(int j= 1;j< = n;+ + j)
for(int k= 1;k< = n;+ + k)
C.A[i][j]+ = A.A[i][k]* B.A[k][j],C.A[i][j];
return C;
}
Mat qpow(Mat a,int b){
Mat tmp= a;
--b;
while(b){
if(b&1) tmp= mul(tmp,a);
b> > = 1;
a= mul(a,a);
}
returntmp;
}
intmain(){
scanf("% lf",&x);
for(int i= 0;i< = 3;+ + i){
B[i]= exp(-x)* pow(x,i)/factorial(i);
}
n= 4,k= 10000;
m_new(ANS,n,n);
ANS.A[1][1]= B[0];
ANS.A[1][2]= B[1];
ANS.A[1][3]= B[2];
ANS.A[1][4]= 1-B[0]-B[1]-B[2];
ANS.A[2][1]= B[0];
ANS.A[2][2]= 0;
ANS.A[2][3]= B[1];
ANS.A[2][4]= 1-B[0]-B[1];
ANS.A[3][1]= 0;
ANS.A[3][2]= B[0];
ANS.A[3][3]= 0;
ANS.A[3][4]= 1-B[0];
ANS.A[4][1]= 0;
ANS.A[4][2]= 0;
ANS.A[4][3]= B[0];
ANS.A[4][4]= 1-B[0];
ANS= qpow(ANS,k);
zz= 200* ANS.A[1][1]+ 250* ANS.A[1][2]+ 400* ANS.A[1][3]+ 600* ANS.A[1][4];
printf("% .3lf ",zz);
return0;
}
```

References

1. Sheldon M. Ross. Applied stochastic processes: an introduction to probabilistic models Beijing: People's Posts and Telecommunications Press 2016: 3.