Analysis of the Calculation of Mathematical Expectations

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Abstract: Mathematical expectations are the most commonly used feature numbers in probability theory and mathematical statistics, and there are various methods for calculating mathematical expectations. This paper systematically studies the calculation methods of mathematical expectations, mainly involving the calculation of mathematical expectations by using distribution functions, characteristic functions, and Moment Generating Function, and also involves the techniques of calculating mathematical expectations such as conditional mathematical expectation formulas, heavy expectation formulas, and mathematical expectation property methods.

Keywords: Mathematical Expectations; Feature Functions; Moment Generating Function; Conditional Mathematical Expectations; Heavy Expectation Formula

1. Introduction

Mathematical expectation is the most commonly used feature number in probability theory and mathematical statistics, and mathematical expectation is also called mean, which is calculated by using the idea of weighted averaging algorithm. Mathematical expectations relate to many eigennumbers as well as concepts and calculations in probability theory and mathematical statistics.

variance $DX = E(X - EX)^2$, Mathematical expectation is included in the definition of variance, and in essence variance is the mathematical expectation of a function of random variables, and variance is also a special form of mathematical expectation.

Coefficient of variation $C_v(X) = \frac{\sqrt{DX}}{EX}$, a dimensionless number of features used to describe fluctuations in random variables, is calculated using mathematical expectations.

Conditional mathematical expectations $E(X|Y)$, Heavy expectation formula $E\left[E(X|Y)\right]$, Feature functions $\varphi(t) = E\left(e^{itX}\right)$, Origin moment $E\left(X^k\right)$ and Center moment $E(X - EX)^k$,The calculation of these feature quantities is inseparable from mathematical expectations.

Statistics, the sample mean $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$, The cut-tail mean $\overline{x_\alpha} = \frac{x_{([n\alpha]+1)} + x_{([n\alpha]+2)} + \cdots x_{(n-[n\alpha])}}{n - 2[n\alpha]}$ is also a form of mathematical expectation. Mathematical expectations are the most basic concepts in probability theory and mathematical statistics, and many studies have been done on the calculation of mathematical expectations.

Hou Wen and Gao Yang [1] analyzed a use of conditional mathematical expectations in solving probability problems from a typical miscalculation brought about by mathematical expectation formulas. Wang Fengxiao [2] discussed the application of exponential parent function in mathematical expectation calculation, solved the exponential parent function of common distributions, and used the exponential parent function to solve the mathematical expectation of some common distributions, and the exponential parent function is also called the moment parent function in some books. Li Xiaoyan and Huang Lili [3] discussed some common mathematical expectation calculation methods. Ning Rongjian and Yu Bingsen [4] analyzed the calculation of mathematical expectations of mixed random variables based on distribution functions, mainly studying some problems of mixed random variables. Xiao Wenhua [5] analyzes some common calculation methods and techniques for mathematical expectations. Chen Hongyan and Deng Zhen [6] analyzed the calculation of mathematical expectations of three kinds of random variables: continuous, discrete, and mixed. Wang Guohua [7] In the article Application of Mathematical Expectations to Classical Probability Computing, the use of mathematical expectations to solve the probabilities in classical equations in turn is an example of a practical application of mathematical expectations.

Based on previous research, this paper further studies and summarizes the algorithm of mathematical expectation.

2. Preliminaries

Definition 1[8] (Distribution function) Let X be a random variable and $F(x)=p(X\le x)$ a distribution function, When X is a discrete random variable, the value of X is $x_1,x_2,\cdots,x_n$, and its distribution is listed as $p(X=x_i)=p_i$, $F(x)=\sum_{x_i\le x}p_i$. When X is a continuous random variable, $p(x)$ is its probability density function, $F(x)=\int_{-\infty}^{x}p(t)dt$.

Definition 2[8] (Feature functions) X is a random variable called $\varphi(t)=E(e^{itX}),-\infty<t<+\infty$ a feature function of X. When X is a discrete random variable, $\varphi(t)=E(e^{itX})=\sum_{k=1}^{\infty}e^{itx_k}p_k$.When X is a continuous random variable, $\varphi(t)=E(e^{itX})=\int_{-\infty}^{+\infty}e^{itx}p(x)dx$.

Definition 3[9] (Moment Generating Function) X is a random variable called $M(t)=E(e^{tX}),-\infty<t<+\infty$ as the Moment Generating Function of X.

Lemma 1[8] (Maximal value distribution) $X_1,X_2,\cdots X_n$ is an independent and homogeneously distributed random variable, $Y=\{X_1,X_2,\cdots X_n\}$, $X_i\sim F(x)$.

The distribution function of Y is $F_Y(y)=[F(y)]^n$.

The density function of Y is $P_Y(y)=F_Y'(y)=n[F(y)]^{n-1}p(y)$.

3. Main conclusion

3.1 Use the distribution function to find mathematical expectations

Definition 4[8] Let X be a random variable on $(\Omega,\mathrm{F},\mathrm{P})$, and if $\int_{\Omega}|x|dp<\infty$, then the mathematical expectation of the random variable X is said to exist, denoted EX.

Then there is $EX=\int_{\Omega}xdp$, which is transformed into $EX=\int_{-\infty}^{+\infty}xdF(x)$ by integral, where $F(x)$ is the distribution function.

When X is a continuous random variable and $p(x)$ is a density function, then there is $EX=\int_{-\infty}^{+\infty}xp(x)dx$.

When X is a discrete random variable and p_i is its distribution column, then there is $EX=\sum_{i=1}^{\infty}x_i p_i$.

Theorem 1 When the distribution function of a random variable is $F(x)$ and the mathematical expectation exists, there is $EX=\int_0^{+\infty}\left[1-F(x)\right]dx-\int_{-\infty}^{0}F(x)dx$.

Further there are $EX=\int_0^{+\infty}p(X>x)dx-\int_{-\infty}^{0}\left[1-p(X>x)\right]dx$.

Proof. From $EX=\int_{-\infty}^{+\infty}xp(x)\,dx=\int_{-\infty}^{0}xp(x)\,dx+\int_0^{+\infty}xp(x)\,dx$

$$\int_{-\infty}^{0}xp(x)\,dx=-\int_{-\infty}^{0}\left(\int_x^0 xdy\right)p(x)\,dx=-\int_{-\infty}^{0}\int_{-\infty}^{y}p(x)dxdy=-\int_{-\infty}^{0}F(y)dy$$

$$\int_0^{+\infty}xp(x)\,dx=\int_0^{+\infty}\left(\int_0^x dy\right)p(x)\,dx=\int_0^{+\infty}\left(\int_y^{+\infty}p(x)\,dx\right)dy=\int_0^{+\infty}\left[1-F(y)\right]dy$$

$$EX=\int_0^{+\infty}\left[1-F(x)\right]dx-\int_{-\infty}^{0}F(x)dx$$

$$F(x)=p(X\le x)$$

$$EX=\int_0^{+\infty}p(X>x)dx-\int_{-\infty}^{0}\left[1-p(X>x)\right]dx$$, Proof is complete.

Deduction 1 When the random variable $X\ge 0$, if $\int_0^{+\infty}(1-F(x))<\infty$, there is $EX=\int_0^{+\infty}\left[1-F(x)\right]dx$ at this time, and further there is $EX=\int_0^{+\infty}p(X>x)dx$.

These definitions and formulas make it easy to solve the mathematical expectations of random variables.

Example 1 $X\sim U(-a,b),a>0,b>0.$ Solving EX.

Solve $X\sim U(-a,b),a>0,b>0.$ $p(x)=\frac{1}{b+a},-a<x<b.$

$$EX=\int_{-\infty}^{+\infty}xp(x)dx=\int_{-a}^{b}x\frac{1}{b+a}dx=\frac{1}{2}(b-a).$$

It can also be solved using theorem 1 $F(x)=\begin{cases}0,x\le -a\\ \frac{x+a}{b+a},-a<x\le b\\ 1,x>b\end{cases}$,

$$EX=\int_0^{+\infty}\left[1-F(x)\right]dx-\int_{-\infty}^{0}F(x)dx=\int_0^b\frac{b-x}{b+a}dx-\int_{-a}^{0}\frac{x+a}{b+a}dx=\frac{1}{2}(b-a).$$

Different methods and conclusions are consistent, of course, it is easier to use definitions here.

3.2 Use characteristic functions to find mathematical expectations

Theorem 2 X is a random variable called $\varphi(t)=E\left(e^{itX}\right),-\infty<t<+\infty$ a characteristic function of X, then there is $EX = \frac{\varphi'(0)}{i}$, where $i^2 = -1$.

Proof. Take a continuous random variable as an example to prove it.

$$\varphi(t) = E\left(e^{itX}\right) = \int_{-\infty}^{+\infty} e^{itx} p(x) dx$$

$$\varphi'(t) = \int_{-\infty}^{+\infty} ixe^{itx} p(x) dx \text{ , } \varphi'(t) = \int_{-\infty}^{+\infty} ixe^{0} p(x) dx = \int_{-\infty}^{+\infty} ixp(x) dx = iEX \text{ , } EX = \frac{\varphi'(0)}{i}$$, Proof is complete.

Example 2 $X \sim \exp(\lambda), \lambda > 0.$ seeking EX

Solve $X \sim \exp(\lambda), \lambda > 0.$ $p(x) = \begin{cases} \lambda e^{-\lambda x}, x > 0 \\ 0, x \le 0. \end{cases}$

$$\varphi(t) = \int_0^{\infty} e^{itx} \lambda e^{-\lambda x} dx = \lambda \left\{ \int_0^{\infty} \cos(tx) e^{-\lambda x} dx + \int_0^{\infty} \sin(tx) e^{-\lambda x} dx \right\}$$

$$= \lambda \left\{ \frac{\lambda}{\lambda^2 + t^2} + i \frac{t}{\lambda^2 + t^2} \right\} = \left(1 - \frac{it}{\lambda}\right)^{-1}$$

Euler's formula in complex functions is used in the above integrals $e^{itx} = \cos(tx) + i \sin(tx)$

exploit $\varphi'(t) = \left(\frac{i}{\lambda}\right)\left(1 - \frac{it}{\lambda}\right)^{-2}$, $\varphi'(0) = \frac{i}{\lambda}$, $EX = \frac{1}{\lambda}$.

Using characteristic functions to solve mathematical expectations requires knowing the characteristic functions of common distributions, and the characteristic functions of common distributions can be solved using distribution functions.

3.3 Use the moment generating function to find mathematical expectations

Theorem 3 X is a random variable called $M(t) = E\left(e^{tX}\right), -\infty < t < +\infty$ as the moment generating function of X. Then there is $EX = M'(0)$.

Proof Take a continuous random variable as an example to prove it.

$$M(t) = E\left(e^{tX}\right) = \int_{-\infty}^{+\infty} e^{tx} p(x) dx ,$$

$$M'(t) = \int_{-\infty}^{+\infty} xe^{tx} p(x) dx \text{ , } M'(0) = \int_{-\infty}^{+\infty} xe^{0} p(x) dx = \int_{-\infty}^{+\infty} xp(x) dx = EX$$,,Proof is complete.

Example 3 $X \sim \exp(\lambda), \lambda > 0.$ seeking EX

Solve By referring to the solution method of the feature function, the moment generating function of the exponential distribution can be found $M(t) = \frac{\lambda}{t - \lambda}$, $M'(t) = \frac{\lambda}{(t - \lambda)^2}$, $M'(0) = \frac{\lambda}{(0 - \lambda)^2} = \frac{1}{\lambda}$, $EX = \frac{1}{\lambda}$.

The method of the moment generating function is similar to the characteristic function, which needs to first find the corresponding moment generating function, and then use the corresponding formula to solve the mathematical expectation. However, the moment generating function is a real number function, and the characteristic function is a complex number function, and they still have their own advantages for use.

3.4 Conditional mathematical expectation method and heavy expectation formula method solve mathematical expectation

Definition 5 [8] Conditional mathematical expectations

Call $E(X|Y=y)=\begin{cases}\sum_i x_i p(X=x_i|Y=y)\\ \int_{-\infty}^{+\infty} xp(x|y)dx\end{cases}$ a conditional mathematical expectation.

$E(X|Y=y)=\sum_i x_i p(X=x_i|Y=y)$, where (X,Y) is a two-dimensional discrete random variable.

$E(X|Y=y)=\int_{-\infty}^{+\infty} xp(x|y)dx$, where (X,Y) is a two-dimensional continuous random variable.

Conditional mathematics expects $E(X|Y=y)$ to be a function of y, not constant, and can be denoted $g(y)=E(X|Y=y)$.

If the specific expression of $g(y)=E(X|Y=y)$ can be found by statistical means and other means, the expected value at this time can be calculated after y takes a certain value.

Example 4 X represents the height of adults, EX represents the average height of adults, and Y represents adult foot length, and biological studies have shown that height and foot length are related.

The average height of adults whose foot length is y is expressed by $E(X|Y=y)$, and the result is $E(X|Y=y)=6.876y$ after statistical research. If a foot length of 25.3 cm is measured in a field, it can be deduced from this formula that the height of the person is about 174 cm.

After all, the expectation obtained by conditional mathematical expectation is a function, and the use of direct expectation often requires the use of statistics to find its functional analytic formula, so more often the use of conditional mathematical expectations is a deeper conclusion and expectation formula.

Let (X,Y) be a two-dimensional random variable and EX exists, then there is $EX=E\left[E(X|Y)\right]$.

If Y is a discrete random variable, then $EX=\sum_j E(X|Y=y_j)p(Y=y_j)$.

If Y is a continuous random variable, then $EX=\int_{-\infty}^{+\infty}\sum_j E(X|Y=y)p_Y(y)dy$.

For some cases, it is difficult to write the distribution column or distribution density function directly, and it is often impossible to directly solve the mathematical expectations. At this time, the conditional mathematical expectation under the conditional distribution can be found first, and then the mathematical expectation can be solved based on the conditional mathematical expectation.

Example 5 If there are n ball numbered $1,2,\cdots,n$ in the pocket, take any 1 ball from them, and if it is the number 1 ball, you will get one point and stop touching the ball. If the i ball is taken, where i is given, $i\geq 2$ points are awarded, and the ball is put back, the ball is touched again, and so on, the average total score obtained.

Solve Taking X as the total score obtained, then the value of X is

$1,2+1,3+1,\cdots,n+1,2+2+1,\cdots$, and it is difficult to write the distribution column of X, and it is impossible to directly find EX. Note Y for this as the number that was first retrieved. Then there is $p(Y=1)=p(Y=2)=\cdots=p(Y=n)=\dfrac{1}{n}$.There is $E(X|Y=1)=1$ again, and when $i\geq 2$,

$E(X|Y=i)=i+E(X)$.

So there is $E(X)=\sum_{i=1}^{n}E(X|Y=i)p(|Y=i)=\frac{1}{n}\left[1+2+\cdots+n+(n-1)EX\right]$

The solution is available $E(X)=\frac{n(n+1)}{2}$.

There are many cases where distribution columns like this are difficult to write, and this also applies to continuous random variables. When it is difficult to write a specific analytical form of distribution, the re-expectation formula can be used to solve this problem that can first find conditional expectations, and then use conditional expectations to solve mathematical expectations.

3.5 Mathematical expectation solution of functions of random variables

For one-dimensional random variables, $Y=g(X)$, solving $EY=E\left[g(X)\right]$ can directly solve such problems by using the formula, and there is generally no difficulty, for example, the variance in the case of ordinary one-dimensional is solved by using the mathematical expectation solution formula of the function of random variables.

However, for the function of multivariate random variables, if it is some relatively simple basic elementary function, the mathematical expectation of the random variable function can be found using Equation

$$EZ=\begin{cases}\sum_i\sum_j g(x_i,y_j)\,p(X=x_i,Y=y_j),\\ \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}g(x,y)\,p(x,y)\,dxdy.\end{cases}$$

. However, for some complex functions, some special functions, the use of formulas often cannot directly solve the problem. At this time, some special computing skills are required to solve the mathematical expectations.

3.5.1 Dimensionality Reduction Method

For some special functions such as the $\max,\min$ function, these functions often cannot give the specific analytic expression of the function, and the mathematical expectation can be calculated by reducing the dimension.

Example 6 Let X_1 and X_2 be independent and homogeneous random variables, and their common distribution is the exponential distribution $\exp(\lambda)$, and find the mathematical expectation of $Y=\max(X_1,X_2)$.

Solve $Y=\max(X_1,X_2)$ does not have a specific analytic formula to solve the formula using the mathematical expectation of a multidimensional random variable function.

The density function of $Y=\max(X_1,X_2)$ can be found using lemma 1.

$$p_Y(y)=2\left(1-e^{-\lambda y}\right)\lambda e^{-\lambda y},y>0.$$

$$EY=E\left[\max(X_1,X_2)\right]=\int_0^{\infty}yp_Y(y)dy=\int_0^{\infty}2y\left(1-e^{-\lambda y}\right)\lambda e^{-\lambda y}dy$$

$$=2\int_0^{\infty}ye^{-\lambda y}d(\lambda y)-\int_0^{\infty}ye^{-2\lambda y}d(2\lambda y)=\frac{2}{\lambda}\int_0^{\infty}ue^{-u}du-\frac{1}{2\lambda}\int_0^{\infty}ve^{-v}dv$$

$$=\frac{2}{\lambda}\Gamma(2)-\frac{1}{2\lambda}\Gamma(2)=\frac{3}{2\lambda}.$$

The dimensionality reduction method is generally suitable for functions where multiple dimensions become one-dimensional. When the function is a multi-dimensional to one-dimensional function, the function can actually be solved by the mathematical expectation formula of the multi-dimensional random variable function when it is relatively simple, and the dimensionality reduction method here can be considered when the function is more complex.

3.5.2 Polar coordinate transformation method

For some more complex elementary functions, the mathematical expectation of directly calculating the function of random variables is not easy to calculate, and can be calculated by polar transformation.

Example 7 Let the random variable X,Y be independent of each other and follow the standard normal distribution $N(0,1)$ to find the mathematical expectation of $Z=\sqrt{X^2+Y^2}$.

Solve According to the question, the density function with $Z=\sqrt{X^2+Y^2}$ is

$$p(x,y)=\frac{1}{2\pi}e^{-\frac{x^2+y^2}{2}},-\infty<x,y<+\infty$$

Then $EZ=E\left(\sqrt{X^2+Y^2}\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{X^2+Y^2}\,p(x,y)dxdy$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{X^2+Y^2}\,\frac{1}{2\pi}e^{-\frac{x^2+y^2}{2}}dxdy,$$

Let $x=r\cos\theta, y=r\sin\theta,$ have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{X^2+Y^2}\,\frac{1}{2\pi}e^{-\frac{x^2+y^2}{2}}dxdy \quad =\int_{0}^{+\infty}\int_{0}^{2\pi}r\frac{1}{2\pi}e^{-\frac{r^2}{2}}rd\theta dr \quad =\int_{0}^{+\infty}re^{-\frac{r^2}{2}}rdr \quad =r\left(-e^{-\frac{r^2}{2}}\right)\Big|_{0}^{+\infty}+\int_{0}^{+\infty}e^{-\frac{r^2}{2}}rdr$$

$$=\frac{\sqrt{2\pi}}{2}\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{r^2}{2}}dr=\frac{\sqrt{2\pi}}{2}$$

The use of polar transformation is generally not very common, and the use of polar transformation method to solve mathematical expectations is mainly based on whether the specific function is suitable for polar transformation.

3.6 Differential derivation method

For some distributions, the probability distribution contains parameters, and the mathematical expectation of random variables can be solved using the method of itemized differentiation.

Example 8 Let the random variable X obey the exponential distribution $\exp(\lambda)$ and find its mathematical expectation EX.

Solve $X\sim\exp(\lambda)$, Its density function can be written out $p(x)=\begin{cases}\lambda e^{-\lambda x}, x\geq 0\\ 0, x<0\end{cases}$,

$EX=\int_{-\infty}^{+\infty}xp(x)dx=\int_{-\infty}^{+\infty}x\lambda e^{-\lambda x}dx$, The integral here can be calculated using the distribution integration method. But it can also be calculated using the technique of derivation. According to regularity there is $1=\int_{-\infty}^{+\infty}p(x)dx$,

For exponential distributions there is $1=\int_{0}^{+\infty}\lambda e^{-\lambda x}dx$. First, the derivation of parameter λ on both sides can have

$\int_0^{+\infty}\left(e^{-\lambda x}-x\lambda e^{-\lambda x}\right)dx=\int_0^{+\infty}e^{-\lambda x}dx-\int_0^{+\infty}xe^{-\lambda x}dx=-\frac{1}{\lambda}\Big|_0^{+\infty}-EX=0$, The solution is available $EX=\frac{1}{\lambda}$.

The original calculation required the method of integration, and the differential derivation method can simplify the operation. In the solution of the mathematical expectation of geometric distribution, the differential derivation method can also be used.

Example 9 Let the random variable X obey the exponential distribution $Ge(p)$ and find its mathematical expectation EX.

Solve $X\sim Ge(p)$, You can write out its distribution column

$p(X=k)=pq^{k-1},0<p<1,q=1-p,k=1,2,\cdots$ $EX=\sum_{k=1}^{+\infty}kpq^{k-1}=p\sum_{k=1}^{+\infty}kq^{k-1}=p\sum_{k=1}^{+\infty}\frac{dq^k}{dq}$.

Differential derivation is applied to mathematical aspirations, where some integrals or other calculations can be converted into differential form.

3.7 Special formula method

For the solution of some mathematical expectations of distributions, some special formulas are used to calculate the final expected value.

Example 10 $X\sim p(\lambda),\lambda>0.$ seeking EX

Solve $EX=\sum_{k=0}^{\infty}k\frac{\lambda^k}{k!}e^{-\lambda}=\lambda e^{-\lambda}\sum_{k=1}^{\infty}\frac{\lambda^{k-1}}{(k-1)!}=\lambda e^{-\lambda}e^{\lambda}=\lambda$

This is where Taylor expands is used $e^{\lambda}=\sum_{k=1}^{\infty}\frac{\lambda^{k-1}}{(k-1)!}$.

There are many special formulas like this, such as $\sqrt{2\pi}=\int_{-\infty}^{+\infty}e^{-\frac{x^2}{2}}dx$ when using definitions to solve the mathematical expectation of a normal distribution, and $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$ in gamma distributions.

3.8 Mathematical expectancy property method

For the solution of some distributed mathematical expectations, it is sometimes much easier to solve them using some properties of mathematical expectations than to solve them directly, and here it is analyzed by solving the mathematical expectations of pairing problems in probability theory.

Commonly used to solve for the properties of expectations and expectations equal to the sum of expectations.

then $E(X_1+X_2+\cdots+X_n)=E(X_1)+E(X_2)+\cdots+E(X_n)$.

Example 11 Mathematical expectations for pairing problems. At a party with Person n, each person brought a gift, different gifts, and during the party, each person randomly selected one of the n gifts put together and asked what the mathematical expectation of X was for the number of people who got their gifts.

Solve In his paper [10], Kuang Nenghui used definitions and formulas to solve the mathematical expectation of the pairing problem, which is more complex, and the solution to this problem is generally solved by using the properties of mathematical expectation.

Using the properties of mathematical expectations to solve for the introduction of a new random variable $X_i = \begin{cases} 1, \\ 0. \end{cases}$, where $X_i = 1$ means that Person i is taking his own gift, where $X_i = 0$ means that Person i is taking someone else's gift, and $i = 0, 1, 2, \cdots n$. Then there is $X = \sum_{i=1}^{n} X_i$, $p(X_i) = \frac{1}{n}, i = 0, 1, 2, \cdots$. so $E(X_i) = \frac{1}{n}$, $EX = E\left(\sum_{i=1}^{n} X_i\right) = 1$.

The properties of mathematical expectations are not limited to this one in the above examples, but other properties of mathematical expectations may also be used to solve other mathematical expectations.

3.9 Symmetric distribution method

When the distribution column, distribution density function, description of the distribution, etc. have symmetry, symmetry can be used to calculate the mathematical expectation of some problems. The concept of symmetry has a wide range of applications in probability theory, and it is difficult to write the specific form of its distribution for some problems, or it is difficult to calculate the integral, at which point if the distribution has symmetry, its mathematical expectation is the symmetry center of its value.

Example 12 There is a stack of poker, a total of n cards, of which there are three K's, shuffle the cards randomly, and then take the cards from the pile from top to bottom until they turn over to the second K, with the random variable X as the number of cards turned, find EX.

Solve X is the number of cards turned from top to bottom, Y is the number of cards turned from bottom to top, and it can be seen from the symmetry that X and Y have the same distribution, then there is $EX = EY$, and $X + Y = n + 1$, using the property of mathematical expectations, there is $E(X+Y) = E(X) + E(Y) = n + 1$, $EX = EY = \frac{n+1}{2}$.

Distribution symmetry is a special case, and its mathematical expectation is often solved by using symmetry.

Conclusions

This paper studies the solution methods of mathematical expectations, and systematically summarizes and summarizes the common mathematical expectation solving methods. There are mainly common formula methods for solving mathematical expectations by using distribution functions, distribution columns and distribution density functions, methods for solving mathematical expectations using characteristic functions and moment mother functions, and methods for calculating mathematical expectations by using some special formulas. Relying on the multi-dimensional distribution, the methods of calculating mathematical expectations such as conditional mathematical expectation formula, heavy expectation formula, and the nature of mathematical expectation were analyzed. In addition, the symmetric distribution method for symmetric distribution cases in mathematical expectation solving and the item-by-item differential method using calculus techniques are analyzed.

These conclusions are of great significance for the study of mathematical expectation solving methods. It is of guiding significance for continuing to think about the solution methods of other numerical features.

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