

Performance Evaluation of Chinese Non-profit Organizations under Uncertain Environment

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Abstract: In recent years, China's nonprofit organizations (NPO) have developed rapidly. The results show that there is a large gap between the efficiency evaluation results under uncertainty and those under certainty. According to the efficiency evaluation results of this paper, we put forward different improvement suggestions for different NPO to improve the management level.

Keywords: NPO; Robust optimization; Data envelopment analysis (DEA); Uncertainty set; Interval DEA

1. Introduction

In recent years, China's NPO have entered a period of rapid development. But the rapid development of NPO has also brought a series of problems. Some NPO have problems such as opaque finance and non-standard business. Even for those large-scale and mature NPO, there are still big management problems. How to effectively manage non-profit organizations in such an era of high uncertainty is an urgent problem to be solved.

When it comes to sustainability and management, the word 'efficiency' is always inseparable from them. Previous studies defined efficiency as 'ability to turn inputs into outputs' (Luksetich & Hughes, 1997)^[1]. The classification of efficiency is also various, such as transactional efficiency (Coupet & McWilliams, 2017)^[2], distributive efficiency (Zerbe, 2002)^[3], productive efficiency (Miragaia, Brito, & Ferreira, 2016)^[4]. Although NPO do not aim at making profits, their sustainable development is inseparable from the support of funds. The world is full of uncertainty. Major profit-making organizations are actively adapting to this new environment, NPO is no exception. Most obviously, NPO's source of funds will be affected, resulting in an impact on its various capital expenditures. Previous studies have assumed that all the data are known. To solve the problem mentioned above, we introduce robust optimization which was first proposed by Soyster (1973)^[5]. Then, Ben-Tal and Nemirovski (1998; 1999; 2000)^[6-8] extended the RO theory and proposed new robust model which based on ellipsoidal uncertainty sets. Subsequently, Bertsimas and Sim (2003; 2004; 2006)^[9-11] developed a robust optimization approach which based on polyhedral uncertainty sets.

In our paper, we proposed the robust data envelopment analysis (RDEA) model which based on an uncertainty sets: Box uncertainty set. Our contributions are as follows:

(1) We proposed robust CCR model and robust BCC model to measure the efficiency of NPO in uncertainty environment. (2) We find that with the change of the degree of uncertainty, the efficiency value and efficiency ranking of NPO may change. (3) We put forward targeted suggestions for NPO with different performances from the micro and macro perspectives.

2. Data envelopment analysis model

Data envelopment analysis (DEA) is an efficient nonparametric method proposed by Charnes et al. (1978) to measure the relative efficiency. Supposed we have n homogeneous decision making units (DMUs). For each DMU _{j} , it consumes m inputs that denoted by x_{ij} and produces s outputs which denoted by y_{rj} . Then we obtain the following model:

$$\begin{aligned}
\max \theta &= \frac{\sum_{r=1}^s u_r y_{ro}}{\sum_{i=1}^m v_i x_{io}} \\
s.t. \quad &\frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}}, \forall j \\
&u_r, v_i \geq 0
\end{aligned} \tag{1}$$

Obviously, model (1) is a fractional programming. Charnes-Cooper transformation can be used to obtain a linear programming (Charnes 1978):

$$\begin{aligned}
\max \theta &= \sum_{r=1}^s u_r y_{ro} \\
s.t. \quad &\sum_{i=1}^m v_i x_{io} \leq 1 \\
&\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad \forall j \\
&u_r, v_i \geq 0
\end{aligned} \tag{2}$$

The model mentioned above is based on the assumption of constant return to scale (CRS), which called CCR model. After that, Charnes and Cooper (1984) proposed a BCC model based on variable return to scale (VRS).

$$\begin{aligned}
\max \theta &= \sum_{r=1}^s u_r y_{ro} + w \\
s.t. \quad &\sum_{i=1}^m v_i x_{io} \leq 1 \\
&\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + w \leq 0, \quad \forall j \\
&u_r, v_i \geq 0
\end{aligned} \tag{3}$$

We use “ \leq ” in DEA model instead of “ $=$ ” in the standard DEA model. The reason why we make this change is to avoid any infeasibility (Ben-Tal 2009)

3. Robust dea model

in model (2) and (3), decision variables and parameters are all deterministic. But in a real-world scenario, this is likely to lead to mistakes when uncertain factors exist. Section 2 propose BCC and CCR models, which based on deterministic assumption. We apply robust optimization to data envelopment analysis to deal with uncertain situation.

Robust optimization is an approach which seeking the optimal solution in the worst case. we consider two uncertainty sets to describe the uncertainty. At the same time, the uncertainty of the outputs are caused by a variety of emergencies. So we consider multiple uncertainty factors in stead of one uncertainty factor.

For the output, it consists of two parts. The first part is determinate value, the other part is uncertainty value. We express the uncertainty as follows

$$U = \{y_{ro}^D = y_{ro} + \sum_{l=1}^L y_{rol}^F \xi_l, y_{rj}^D = y_{rj} + \sum_{l=1}^L y_{rjl}^F \xi_l, \xi_l \in Z \} \quad (4)$$

y_{ro} and y_{rj} denotes the outputs in the deterministic situation, y_{rol}^F and y_{rjl}^F are outputs fluctuation caused by different uncertainty factor. ξ_l represents uncertainty factor.

Finally, the following Equations can be obtained:

RCCR (Robust CCR)

$$\begin{aligned} & \max \theta \\ \text{s.t. } & \theta - \sum_{r=1}^s u_r (y_{ro} + \sum_{l=1}^L y_{rol}^F \xi_l) \leq 0 \\ & \sum_{i=1}^m v_i x_i \leq 1 \\ & \sum_{r=1}^s u_r (\sum_{l=1}^L y_{rjl}^F \xi_l) - \sum_{i=1}^m v_i x_{ij} \leq 0, \forall j \\ & u_r, v_i \geq 0 \end{aligned} \quad (5)$$

RBCC (Robust BCC):

$$\begin{aligned} & \max \theta \\ \text{s.t. } & \theta - \sum_{r=1}^s u_r (y_{ro} + \sum_{l=1}^L y_{rol}^F \xi_l) - w \leq 0 \\ & \sum_{i=1}^m v_i x_i \leq 1 \\ & \sum_{r=1}^s u_r (\sum_{l=1}^L y_{rjl}^F \xi_l) - \sum_{i=1}^m v_i x_{ij} + w \leq 0, \forall j \\ & u_r, v_i \geq 0 \end{aligned} \quad (6)$$

3.1 Robust model based on box uncertainty set

For RBCC model, we consider the most simple uncertainty box-set uncertainty set first.

Proposition 1. We define the box uncertainty set as Z^B

$$Z^B = \{\xi \in R^L : \|\xi\|_\infty \leq \Theta\}$$

Then the RCCR model can be constructed as

$$\begin{aligned} & \max \theta \\ \text{s.t. } & \theta + \Theta \sum_{l=1}^L \sum_{r=1}^s u_r y_{rol}^F - \sum_{r=1}^s u_r y_{ro} \leq 0 \\ & \sum_{i=1}^m v_i x_{io} \leq 1 \\ & \Theta \sum_{l=1}^L \sum_{r=1}^s u_r y_{rjl}^F + \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \forall j \\ & u_r, v_i \geq 0 \end{aligned} \quad (7)$$

Proof. According to the form of box uncertainty set, we rewrite the first constraint of model (6) as

$$\theta - \sum_{r=1}^s u_r y_{ro} - \sum_{l=1}^L \sum_{r=1}^s u_r y_{rol}^F \xi_l \leq 0, \forall \{\xi : \|\xi\|_{\infty} \leq \Theta\} \quad (8)$$

It is equivalent to

$$\min_{-\Theta_l \leq \xi_l \leq \Theta_l} \sum_{l=1}^L \sum_{r=1}^s u_r y_{rol}^F \xi_l \geq \theta - \sum_{r=1}^s u_r y_{ro}, \forall \{\xi : \|\xi\|_{\infty} \leq \Theta\} \quad (9)$$

The minimum value of the left-hand side of constraint (9) is

$$-\Theta \sum_{l=1}^L \sum_{r=1}^s u_r y_{rol}^F$$

The explicit form of the first constraint of model (5) can be obtained.

$$\theta + \Theta \sum_{l=1}^L \sum_{r=1}^s u_r y_{rol}^F - \sum_{r=1}^s u_r y_{ro} \leq 0$$

The same procedure may be easily adapted to obtain the explicit form of the third constraint of model(6)

$$\Theta \sum_{l=1}^L \sum_{r=1}^s u_r y_{rjl}^F + \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0$$

Similar to the procedures of obtaining the RCCR(box-uncertainty)model,we get RBCC model

$\max \theta$

$$s.t. \quad \theta + \Theta \sum_{l=1}^L \sum_{r=1}^s u_r y_{rol}^F - \sum_{r=1}^s u_r y_{ro} - w \leq 0$$

$$\sum_{i=1}^m v_i x_{io} \leq 1 \quad (10)$$

$$\Theta \sum_{l=1}^L \sum_{r=1}^s u_r y_{rjl}^F + \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + w \leq 0, \forall j$$

$$u_r, v_i \geq 0$$

4. Data

4.1 Data resource

We selected seven representative NPO, which are large-scale and standardized head charitable foundations in China. They speak for different interest groups. The data are from the balance sheets and business activity statements of various foundations. All the data are public and can be downloaded. For the convenience of addressing, we named them DMU1 (decision making unit), DMU2, DMU3, DMU4, DM5, DMU6 and DMU7.

4.2 Inputs and outputs

The choice of input or output is important for DEA and RDEA method. The efficiency value measures the operating of NPO and reflects the ability of NPO to operate for a long time.

The input is the financial expenditure and liabilities of NPO, the first selected input indicator is accounts payable. Accounts payable is a typical liability arising from NPO, because government organizations generally do not involve profit-making activities except official business, their purchase behavior is guaranteed by financial funds. There are few accounts payable but unpaid. The other two important input indicators are wages and taxes payable. They show the expenditure of NPO on staff salaries and taxes payable to the government.

Corresponding to the input are NPO's financial revenue and assets. We choose monetary assets, short-term investment and fixed

assets as output indicators. Chinese NPO make less long-term investment and prefer short-term flexible investment. Therefore, monetary assets and short-term investment are good indicators. Fixed assets refer to non monetary assets held by NPO for a long time, which shows the financial sustainability of NPO. We select seven representative NPO organizations, sound financial data, standardized operation and mature management. All data are from the annual audit report issued in 2020. Table 1 presents descriptive data for seven DMUs.

Table 1 Descriptive data of 7 dmus.

	Mean	Standard deviation	Maximum	Minimum
Inputs				
Accounts payable	1022	797	2334	89
Wages payable	1621	1937	4657	537
Tax payable	199	134	428	8
Outputs				
Monetary Asset	196658	167530	426772	23587
Short-term investment	355567	304080	895471	102000
Fixed assets	5258	5301	16903	1708

5. Results

5.1 Dea results

The efficiency value calculated by the CCR model is called the comprehensive technical efficiency value (TE). The calculation of the TE does not take into account the impact of the scale change of the DMU. The efficiency value calculated by BCC model is called pure technical efficiency value (PTE), which considers the case of variable scale. The scale efficiency value (SE) = TE / PTE, which measures the gap between the existing and the optimal scale. When TE, PTE and SE are equal to 1, we call it DEA efficient. When they are less than 1, we call it DEA inefficient. Table 2 shows the calculation results of BCC model and CCR model under certainty. From the results of TE, the average value is 0.81, which is generally at a high efficiency level. The efficiency values of DMU2, DMU3, DMU4, DMU6 and DMU7 all reach 1, which is an effective DMU of DEA. The efficiency values of DMU5 and DMU1 are low, which indicates that their financial operation ability is insufficient, and there is a large gap with the top DMUs, indicating that there is a significant internal business gap among China's large NPO. From the results of PTE and SE, the efficiency values of DMU2, DMU3, DMU4, DMU6 and DMU7 are all 1, which indicates that they are quite excellent in management level, their scale is in the best scale state. For DMU5 and DMU1, there is a large room for improvement in their PTE, which requires them to make great improvements in the management of NPO. Their SE is also invalid, which shows that DMU5 and DMU1 can achieve the effectiveness of TE by improving their SE.

Table 2 dea results

	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	Mean
TE	0.299	1.000	1.000	1.000	0.372	1.000	1.000	0.810
PTE	0.424	1.000	1.000	1.000	0.416	1.000	1.000	0.834
SE	0.705	1.000	1.000	1.000	0.894	1.000	1.000	0.943

5.2 Robust DEA results based on box uncertainty set

The results of the previous calculation are the results in the deterministic case. We will calculate the efficiency value based on RCCR model and RBCC model in the uncertain case. In robust CCR and BCC model we consider the uncertainty of the output, but also the influence of different uncertain factors on the outputs. We consider the impact of policy change and economic shocks on output. We set $l = 2$ that means we consider two uncertain factors which affect the outputs. We suppose $y_{rjl}^F = 0.02y_{rj}$, $y_{rol}^F = 0.02y_{rj}$, the uncertainty parameter range from 1 to 3, which represents different degrees of uncertainty. From the previous parameter setting results, we can know the output disturbance value range from 0 to 0.1.

When the uncertainty set is a box uncertainty set, the calculated efficiency value is shown in Table 3. We can look at the overall change from the results of the average value. With the increase of parameters Θ (i.e. the increase of the degree of uncertainty), TE, PTE and SE all decrease, but the rate of decline is different. The scale of TE and SE decreases rapidly and PTE decreases slowly, which shows that the increase of uncertain factors has a great impact on the SE and TE of NPO, and has a small impact on the PTE. For DMU individuals, the increase of uncertain factors may bring different results. For DMU2, DMU3, DMU6 and DMU7, Even if the parameter Θ increases from 1 to 3, their PTE value remains valid, and the degree of uncertainty has no impact on their PTE, their financial management ability and technical level are not affected, the reduction of their TE value is caused by the reduction of SE value. It is worth noting that DMU4, in the case of certainty, when the parameter Θ is equal to 0, its PTE value is 1. When the parameter Θ increases

from 0 to 3, PTE shows a downward trend. This means that DMU4's financial management ability will be impacted by uncertain factors. In an uncertain environment, it will change from effective PTE to ineffective PTE. For DMU5, its PTE value is also quite stable, but it has always been at a low level. Coupled with the impact of uncertain factors on its SE, the TE value is in a state of linear decline. For DMU1, its SE and PTE are greatly affected by uncertain factors. When they decline at the same time, the TE decreases significantly.

Table 3 rdea results under box uncertainty

		DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	Mean
$\Theta=1$	TE	0.276	0.923	0.923	0.923	0.344	0.923	0.923	0.748
	PTE	0.415	1	1	0.952	0.416	1	1	0.826
	SE	0.664	0.923	0.923	0.97	0.827	0.923	0.923	0.879
$\Theta=2$	TE	0.254	0.852	0.852	0.852	0.317	0.852	0.852	0.690
	PTE	0.408	1	1	0.907	0.416	1	1	0.819
	SE	0.623	0.852	0.852	0.939	0.762	0.852	0.852	0.819
$\Theta=3$	TE	0.235	0.786	0.786	0.786	0.293	0.786	0.786	0.637
	PTE	0.408	1	1	0.865	0.416	1	1	0.813
	SE	0.576	0.786	0.786	0.909	0.704	0.786	0.786	0.762

6. Conclusion

China's NPO has developed rapidly in the last decade. but excessive growth also brings some problems. Due to the large number and different scales of NPOs, the management problem has naturally become an urgent problem to be solved. This paper uses DEA method to measure the management efficiency of NPO organization, and focuses on the financial efficiency of NPO, because the financial efficiency reflects the sustainability and operation of NPO organization. After the outbreak of COVID-19, the whole world is at a high level of uncertainty. Previous studies ignored the impact of uncertain factors on the efficiency of NPO. This paper focuses on the impact of uncertain factors on NPO. We try to build a model that can accurately measure NPO efficiency in uncertain environment. The DEA model is improved by robust optimization method. We try three kinds of uncertain sets: box uncertain set, ellipsoid uncertain set and polyhedron uncertain set. The empirical results show that: (1) our model accurately quantifies the impact of uncertain factors on NPO operating efficiency. Our RBCC model and RCCR model can simulate the degree of uncertainty by controlling uncertain parameters. Some results will change with the change of uncertainty, which is more in line with the situation of the real world. (2) The main reason for maintaining efficient NPOs in a certain environment may not be efficient under the impact of uncertain factors is the change of scale efficiency. (3) the government should increase its support for inefficient NPOs.

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