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The Uniform Dimension of the Level Sets of a Two-Parameter Ornstein-Uhlenbeck Process

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Abstract: In this paper, based on Chen (1992) on the Hölder continous of local times of two-parameter Ornstein-Uhlenbeck process, we study the uniform dimension for the level sets of two-parameter OUP by using local times approach.

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Keywords: Two-parameter OUP; Level sets; Uniform dimension

1. Introduction

Let $\{B_t^d ,t \geq 0\}$ denote a d-dimensional Brownian motion, $d \geq 2$, Kaufman (1969) got the first uniform dimension result for the image sets of $\{B_t^d ,t \geq 0\}$ as follows : for any closed subset $E \subset R^1$, one has $\dim B^d(E) = 2\dim E$ a.s. ; while for a 1-dimensional Brownian motion $\{B_t^1 ,t \geq 0\}$, Kaufman (1985) proved that $\dim B^1(E) = 2\dim E \wedge 1$ a.s., where $\dim E$ denotes the Hausdorff dimension of the set E. Here the exceptional sets are not dependent on E.

As to the level sets of a N-parameter Wiener process, Adler (1978, 1980) by using local times, which is different from the approach of Kaufman, proved that the Hausdorff dimension of the level sets of a N-parameter Wiener process equals $N - \frac{1}{2}$,

with probability one.

In this paper, we study the uniform dimension of the level sets of a two-parameter Ornstein-Uhlenbeck process which was introduced by Wang (1984) as follows:

$$X(t_1,t_2) = e^{-(\alpha_1 t_1+\alpha_2 t_2)}\left(X_0 + \sigma\int_0^{t_1}\int_0^{t_2} e^{-(\alpha_1 a_1+\alpha_2 a_2)} dW(a_1, a_2)\right) \tag{1.1}$$

where $\{W(t_1,t_2),t_1 \geq 0, t_2 \geq 0\}$ is a Brownian sheet, and X_0 is a random variable that is indpendent of W. Similarly, we can define N-parameter d-dimensional Ornsten-Uhlenbeck process (N, d) OUP. When X_0 does not have a normal distribution,

usually $\{X(t_1,t_2),t_1 \geq 0, t_2 \geq 0\}$ does not have the local nondeterminism property. Thus we can not get the uniform dimension results of two-parameter Ornstein-Uhlenbeck process $\{X(t_1,t_2),t_1 \geq 0, t_2 \geq 0\}$ directly from that of local nondeterminism Gaussian random fields.

Wang (1984) studied Markov properties and solved prediction problems for two-parameter Ornstein-Uhlenbeck process $\{X(t_1, t_2), t_1 \geq 0, t_2 \geq 0\}$ as Gaussian Markov random fields. Meanwhile sample path properties, such as Haudorff dimension of image and graph sets, Hölder continous of local times, and interval recurrence property were studied for (N, d) Ornsten-Uhlenbeck process in Chen (1989, 1992), Wang and Chen (1997). The difference between (N, d) Wiener processes and (N, d) Ornstein-Uhlenbeck processes is that (N, d) Wiener processes have scaling and independent increment properties, while Ornstein-Uhlenbeck processes do not have such properties. A surprising difference between (N, d) Wiener process and (N, d) Ornstein-Uhlenbeck process was found in Wang and Chen (1997): for an arbitary pair of positive integers (N, d) and any open set S in R^d, (N, d) Ornstein-Uhlenbeck processes are recurrent to S; while when $d > 2N$, Orey and Pruitt (1973) proved that (N, d) Wiener processes are not interval recurrent but transit.

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