Instructional design of mathematical modeling activity based on PBL theory

Xiangxiang Jiang, Xuejun Zhou, Zhenzhen Chen*
College of Mathematics and Statistics, Huanggang Normal University, Hubei, Huanggang 438000

Abstract: To conduct a successful mathematical modeling activity in high schools is inseparable from effective guidance of the teachers and active participation of the students. Integrating PBL (problem-based learning) theory into the instructional design of mathematical modeling activity may help improve teachers' poor guidance and students' insufficient participation. This chapter adopts a speculative research method, distinguishes the two terms of mathematical modeling and mathematical modeling activity, and discusses the general teaching process of integrating PBL theory into mathematical modeling activities, and applies this process to design a specific case. The teaching idea and method in this chapter can provide reference for the design of mathematical modeling activities.

Key words: mathematical modeling; mathematical modeling activity; PBL theory; instructional design

1. Introduction

As the new curriculum reform in Chinese high schools continues, from 1996 to 2020, the number of entries related to mathematical modeling in the programmatic text of Chinese mathematics curriculum has gradually increased from 0 to 74 (Huang, 2019). Introducing mathematical modeling activities into the classroom is an important measure to further improve mathematics education in China. In this way, high school mathematical modeling activities have attracted more and more attention from teachers and students. However, due to the lack of theoretical support and research, there are still problems such as poor teacher guidance and insufficient student participation.

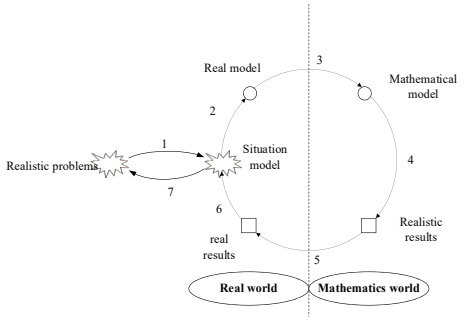
PBL (problem-based learning) is a problem-oriented educational philosophy (Dong and Sun, 2019). Luo (2018) applied the PBL theory to high school mathematics teaching and believed that it could change the current teaching status in China.

By integrating PBL (problem-based learning) theory into the instructional design of mathematical modeling activity, the collision between the two may rub off sparks, allowing students to experience that mathematics is not only a discipline, but also a tool. In a sense, it also conforms to the principle of "student-centered, teacher-led" required by the high school mathematics Curriculum Standards in China Therefore, we can realize that the integration of PBL concept and mathematical modeling activity course is complementary to each other, which can effectively improve the current situation of modeling activity course.

2. Mathematical modeling and mathematical modeling activity

Mathematical modeling is to first understand the situation and abstract it into a situation model, then transform it into a realistic model, and then translate it into a mathematical problem (Blum, 2007). The specific process of mathematical modeling is shown as Figure 1 .

Mathematical modeling activity is a kind of mathematics course type in the high school mathematics Curriculum Standards in China. (MOE of PRC, 2020). The high school mathematical modeling activity is based on the cognitive development of high school students and the existing knowledge and experience, and its difficulty and requirements are lower than the general mathematical modeling (Li, 2022). The high school mathematics Curriculum Standards in China generally believes that mathematical modeling is to use mathematical language to turn the actual problems in real life into mathematical problems, abstract them into mathematical models, and then solve the solution of the mathematical model through operations, reasoning, etc., and then verify the rationality of the mathematical model, and use conclusions to explain an application process of practical problems (see Figure 2).



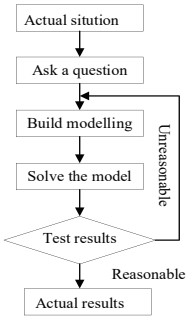


Fig 1 The process of mathematical modeling **Fig 2 The process of mathematical modeling activity**

Both of them are based on practical problems, transforming practical problems into mathematical problems and solving them. However, the problem difficulty of high school mathematical modeling activity is lower than that of mathematical modeling, and it is a practical application problem of non-pure mathematical theory.

3. Teaching process of integrating PBL theory into mathematical modeling activity

Teaching process of integrating PBL theory into mathematical modeling activity can be clarified as follows. First of all, teachers should

connect with practical problems, create situations, design problems, and stimulate students' thinking. Secondly, students communicate in the form of group cooperation after independent thinking, select models and communicate and display. After listening to the views, the group optimizes the model and solves practical problems. Finally, each group submits the work in the form of a report. In the whole process, teachers should evaluate students from all aspects and students should evaluate each other (see Figure 3).

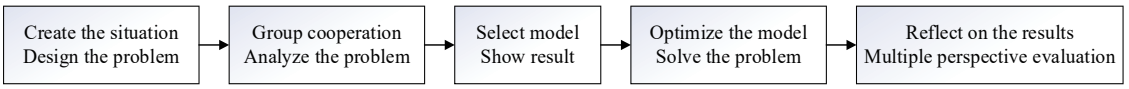


Fig 3 The teaching process of integrating PBL theory into mathematical modeling activity

A case applied instructional design

This section designs the teaching process of establishing function module to solve practical problems - the best drinking time of tea, which selected from Chinese high school mathematics textbook.

The mathematical modeling activity consists of four components, namely, choosing a topic, starting a project, doing a project and completing a project. Topic selection, proposal and solution are all completed by students in their spare time. Herein, we only design the teaching process of doing a project.

(1) Display and report the results

Teachers provide a platform for group representatives to show and summarize the information given and give the factors that affect the taste of tea. And the problem is thrown out: a certain green tea is brewed with water at 85°C, and the best taste is when drinking the tea at a temperature of 60°C. At room temperature of 25°C, how long does it take for the freshly brewed tea to achieve the best taste? After discussion, students, it was appropriate for the third group to measure the tea temperature by thermometer every 1 minute. The data are shown in Table 1.

Table 1 The data of temperature variation with time

Time/min	0	1	2	3	4	5
Temperature/℃	85.00	79.26	75.13	71.08	67.83	64.17

In this process, the teacher guides students to transform practical problems into mathematical problems, and cultivate students' consciousness of observing the world from the perspective of mathematics.

(2) Problem inquiry

The teacher throws questions to make students think.

Question 1: Whether there is a connection between time and water temperature in this set of data? Can the relationship between them be expressed in terms of functions?

Question 2: If it can be expressed by function, can we use the function model we have learned to describe the relationship between time and water temperature? If not, how should it be expressed?

(3) Cooperative discussion

Students know to use the function model but do not know which one to choose. The teacher guides the students to observe with the image method and then choose the appropriate function model. Finally, teachers guide students to guess all the function models learned before according to the distribution law of points: linear function, exponential function and power function.

(4) Model selection

Different groups choose different functional models, almost all in primary, power and exponential functions

In this process, teachers do not limit conditions and give students full space to display, thus cultivating students' divergent thinking as much as possible.

(5) Model calculation

The teacher observes and helps students to find the unknowns, and gives the answers of some groups : the model obtained by the first group is , the model of the second group is , the model obtained in the third group is , and the model obtained in the seventh group is . According to the model of their own calculation, combined with their own assumptions, students reflect on the shortcomings of the model they choose. In the calculation of students, the teacher finds that the function model selected by the first group and the third group is the same, but the results obtained are slightly different due to the different calculation methods. Therefore, the question of which group is more accurate is thrown to cause students to think.

Through discussion, listening to each group of speakers, and combining the assumptions of all parties, students believe that the exponential function model is more appropriate, and that the model of obtained by the third group is more accurate in calculation.

In this process, teachers let students first elaborate their own ideas, communicate with each other, listen to good suggestions, and discuss which model is more suitable. At the same time, students' language expression ability has also been improved, and students' mathematical operation and mathematical modeling literacy have also been developed.

(6) Model optimization

According to the model assumed by the students, the teacher leads the students to use the software Origin to fit the function. The teacher operates it and the students observe and learn the software. Here, only the images of the exponential function model are shown Figure 5. The analytical formula of the exponential function model is .

This process is a difficult part of the problem, which provides students with different teaching tools, making students understand the intuition and superiority of information technology to solve problems. The students can better understand the basic steps of mathematical modeling, and constantly cultivate students' mathematical modeling literacy.

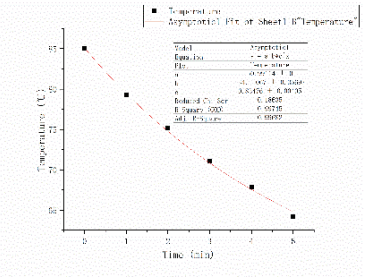


Fig 4 Fitting graph of exponential function

(7) Problem solving

The teacher puts forward the question: At room temperature of 25°C, how long does freshly brewed tea place for optimal taste? Then students think and solve it. The teacher guides students to put into the function fitted by the software to get.

As a result, it takes about 7 minutes to make a cup of tea with the best taste.

In this process, students apply the knowledge they have learned to practice, and experience the application value of mathematics.

(8) Results reflection

The teacher asks the question: Why is the exponential function more fitted? This question guides students' thinking to diverge into other disciplines. The teacher explains: Newton's law of cooling in physics, expressed as a function between temperature and time, describes a process of exponential decay. So the fit degree of exponential function is higher.

In this process,let the students experience the connection between different disciplines.

(9) Multiple evaluation

In the previous links, the teacher makes comments from the aspects of students' thinking, problems solving, communication and so on. During the whole process of students' learning, teachers are observing their performance. After this class, teachers will give students an evaluation form. At the same time, after the end of the study, students should evaluate each member of the group. The evaluation link is not described one by one due to the length of the article.

The process is left for students to finish after class, and a study should be written by the group.

Students will improve and revise the results of previous research. This session is done by students after class.

5. Reflection and summary

In order to incorporate PBL into the design of mathematical modeling activities, the teacher first transforms complex mathematical knowledge into a life problem based on the principle of the area of proximal development according to the students' familiarity with the optimal tea time. Then, on the premise of knowing all possible answers in advance, the teacher guides the students to think about this life question by constantly asking questions. Secondly, in the stage of choosing the mode, the students had different answers, but the teacher did not directly deny them. On the contrary, teachers adhere to the PBL and standardize the classroom by asking questions, which undoubtedly improves the guiding role of teachers and makes teaching vivid and efficient.

Reference:

[1]Huang, J., Lu, X., Wang, Y., &Xu, B. (2019). Mathematical modeling in Mathematics Curriculum Standards in China since the 20th (in Chinese). Journal of Mathematics Education, 28 (3): 18-23.

[2]Dong, Y., & Sun, W. (2019). Research on the Production-based Learning (DoPBL) Model for Promoting Interdisciplinary Learning: Based on the Integrated Perspective of Problem-based Learning and Project-based Learning(in Chinese) . Journal of Distance Education.37(02):81-89.

[3]Luo, Y. (2018). Research on the Implementation Strategy of PBL Teaching Method in High School Mathematics--Taking the Application of Function Teaching as an Example(in Chinese) . Mathematics Teaching Communication.(12):33-34.

Blum, W. (2007) Modelling and application in mathematics education: The 14th ICMI study(pp. 3-32). New ICMI Study.

Ministry of Education of the People's Republic of China. (2020). Ordinary high school mathematics curriculum standards: 2017 edition 2020 revision (in Chinese). Beijing: People 's Education Press.

Ministry of Education of the People ' s Republic of China. (2020). Ordinary high school curriculum plan:2017 edition 2020 revision (in Chinese). Beijing: People 's Education Press.

Li, J. (2022) Research on the current situation and teaching strategies of integrating high school mathematics teaching into mathematical modeling activities(in Chinese) .Southwest University.

This paper is a project fund: Huanggang Education Science Planning Fund in 2022 (2022GB36) "Practical Research on Performance Evaluation in the Teaching of Mathematical Modeling Activities in Senior High Schools".