# Study on three-phase dynamic model of asynchronous motor

*Yan Lu[1], Guangwei Cheng[1], Yuan Zhang[1], Kui Zhou[2]*
1. Shaanxi Railway Institute,School of Urban Rail Transit Engineering, Weinan 714000, China
2. Survey and Design Institute of Mechanical Industry Co., LTD., Xi 'an 710000, China

**Abstract:** Asynchronous motor has a lot of advantages, such as high output efficiency, wide range of speed adjustment, high density, so it has been widely used in many occasions with high dynamic performance requirements, such as electric vehicle manufacturing, industrial robot processing, passenger elevator manufacturing. In this paper, based on the dynamic model of asynchronous motor, the mathematical model of it was derived in detail, and finally obtained the motor flux equation, voltage equation and so on. Focus on the study of how to control the torque and flux of asynchronous motor, and get its control law, for the future asynchronous motor for more in-depth research has obtained a good foundation.

**Key words:** Asynchronous motor; Dynamic model; Flux linkage equation; Torque equation

## 1. Introduction

In recent years, with the rapid development of power electronic products, the performance of asynchronous motors continues to improve, making it more and more widely used occasions. The output performance of the asynchronous motor presents a nonlinear relationship, the coupling between the parameters is strong, and the input variables are more, based on these properties, so that the asynchronous motor can be smoothly adjusted in the specified range of its speed, but the electric vehicle manufacturing, industrial robot processing, passenger elevator manufacturing and other fields of the asynchronous motor dynamic performance requirements are very high, Under the current circumstances, asynchronous motors can not meet the needs of production. If you want to get a mathematical model of high dynamic performance and wide speed regulation range, you need to start from the asynchronous motor itself, get how to control the torque and flux of the asynchronous motor, and more in-depth study how to adjust the speed of the asynchronous motor in a wide range of ways.

## 2. Three-phase dynamic model of asynchronous motor

The dynamic model of asynchronous motor is composed of a number of equations, such as flux equation, voltage equation, torque equation and motion equation, etc. In these equations, flux equation and torque equation are reflected in the form of algebraic equations, while voltage equation and motion equation are presented in the form of differential equations.

2.1 Flux equation

Asynchronous motor has six windings, each winding itself will form a self-inductance flux, the other five windings will affect this flux, thus forming a mutual inductance flux, therefore, the self-inductance flux of each winding plus the mutual inductance flux of the other five windings can be obtained the total flux of this winding, the flux of all windings can be expressed by formula (1) :

$$\begin{bmatrix} \psi_A \\ \psi_B \\ \psi_C \\ \psi_D \\ \psi_E \\ \psi_F \end{bmatrix} = \begin{bmatrix} L_{AA} & L_{AB} & L_{AC} & L_{Aa} & L_{Ab} & L_{Ac} \\ L_{BA} & L_{BB} & L_{BC} & L_{Ba} & L_{Bb} & L_{Bc} \\ L_{CA} & L_{CB} & L_{CC} & L_{Ca} & L_{Cb} & L_{Cc} \\ L_{aA} & L_{aB} & L_{aC} & L_{aa} & L_{ab} & L_{ac} \\ L_{bA} & L_{bB} & L_{bC} & L_{ba} & L_{bb} & L_{bc} \\ L_{cA} & L_{cB} & L_{cC} & L_{ca} & L_{cb} & L_{cc} \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_a \\ i_b \\ i_c \end{bmatrix} \quad (1)$$

Or written as $\boldsymbol{\psi} = \boldsymbol{Li}$

In formula (1), represents the instantaneous change of the phase current through the stator and rotor; $i_A, i_B, i_C, i_a, i_b, i_c$ $\psi_A, \psi_B, \psi_C, \psi_a, \psi_b, \psi_c$ Represents the sum of the self-inductance flux of each phase winding and its mutual inductance flux. $\boldsymbol{L}$ Represents the 6×6 inductance matrix formed by all inductors, in this matrix equation, the parameter is in the diagonal position of the equation, indicating the meaning of each winding on its own inductance value, the remaining parameters in the matrix represent the meaning of each phase winding inductance value induction between each other. $L_{AA}$、$L_{BB}$ $L_{CC}$ $L_{aa}$ $L_{bb}$ $L_{cc}$ The meaning of the stator leakage inductance is loaded in the stator all leakage flux above the inductance value, at the same time, the meaning of the rotor leakage inductance is loaded in the rotor all leakage flux above the inductance value, asynchronous motor each phase winding is symmetrical, so that the leakage inductance value of all windings is also in an equal state. $L_{ls}$ Among all the parameters, the stator mutual inductance corresponds to the highest mutual inductance flux value generated in the stator phase winding, at the same time, the rotor mutual inductance corresponds to the highest mutual inductance flux value generated in the rotor phase winding, all the winding values on the stator and the rotor can be converted to obtain the number of turns of the stator and the rotor is equal. $L_{ms}$ $L_{mr}$ Therefore, it can be obtained. $L_{ms} = L_{mr}$ In this formula, all parameters have been converted, and all are on the stator side. For the convenience of expression, all the upper corner symbol "’" representing the converted is omitted, and the following expressions are the same.

The magnetic flux value generated by each phase winding of asynchronous motor is the mutual inductance flux generated by other windings for the phase winding plus leakage induction flux, so the self-inductance value loaded on each phase winding of the stator is equal to

$$L_{AA} = L_{BB} = L_{CC} = L_{ms} + L_{ls} \qquad (2)$$

Rotor phase self-inductance (3) $L_{aa} = L_{bb} = L_{cc} = L_{ms} + L_{lr}$

The mutual inductance between all windings of asynchronous motor can be divided into two cases: ① the stator A, B, C three-phase winding and rotor A, B, C three-phase winding between the mutual position is fixed, in this case, the mutual inductance between all windings is a constant; ② the position between each phase of stator A, B, C three-phase winding and rotor A, B, C three-phase winding is constantly changing, at this time the mutual inductance value must be expressed by the function equation related to angular displacement.

The first situation is relatively simple, so the first situation is analyzed, from the perspective of spatial position, the phase difference between A, B and C three-phase windings can be used to express, here that the magnetic flux between the air gap is in a sinusoidal distribution state, the mutual inductance value is $\pm\frac{2\pi}{3}$ $L_{ms}\cos\frac{2\pi}{3} = L_{ms}\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}L_{ms}$ $L_{AB} = L_{BA} = L_{CA} = L_{BA} = L_{CB} = L_{AC} = -\frac{1}{2}L_{ms}$

$L_{ab} = L_{bc} = L_{ca} = L_{ba} = L_{cb} = L_{ac} = -\frac{1}{2}L_{ms}$

$$L_{ab} = L_{bc} = L_{ca} = L_{ba} = L_{cb} = L_{ac} = -\frac{1}{2}L_{ms} \qquad (4)$$

Next discuss the second case, stator A, B, C three-phase winding and rotor A, B, C three-phase winding between each phase of the position is constantly changing, can be expressed by the following formula (5) :

$$L_{Ab} = L_{bA} = L_{Bc} = L_{cB} = L_{Ca} = L_{aC} = L_{ms}\cos\left(\theta + \frac{2\pi}{3}\right) \quad L_{Ac} = L_{cA} = L_{Ba} = L_{aB} = L_{Cb} = L_{bC} = L_{ms}\cos\left(\theta - \frac{2\pi}{3}\right) \qquad (5)$$

If the axis of any two phases in the stator and rotor windings of the asynchronous motor is in a state of coincidence, the mutual inductance value between the two phase windings is the maximum mutual inductance value, and the maximum mutual inductance value is expressed. $L_{ms}$

The formula (4), formula (5) and formula (1) combined, you can get the induction motor flux equation, if the block matrix to reflect, can be expressed by formula (6) :

$$\begin{bmatrix} \dot{\boldsymbol{\phi}}_{s} \\ \dot{\boldsymbol{\phi}}_{r} \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_{ss} & \boldsymbol{L}_{sr} \\ \boldsymbol{L}_{rs} & \boldsymbol{L}_{rr} \end{bmatrix} \begin{bmatrix} _{s} \\ _{r} \end{bmatrix} \qquad (6)$$

In formula,; $\boldsymbol{\phi}_{s} = [\psi_A \quad \psi_B \quad \psi_C]^T$ $\boldsymbol{\phi}_{r} = [\psi_a \quad \psi_b \quad \psi_c]^T$ $\boldsymbol{i}_s = [i_A \quad i_B \quad i_C]^T$ $\boldsymbol{i}_r = [i_a \quad i_b \quad i_c]^T$

$$\boldsymbol{L}_{ss} = \begin{bmatrix} L_{ms} + L_{ls} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{ms} + L_{ls} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{ms} + L_{ls} \end{bmatrix} \qquad (7)$$

$$\boldsymbol{L}_{rr} = \begin{bmatrix} L_{ms} + L_{lr} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{ms} + L_{lr} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{ms} + L_{lr} \end{bmatrix} \qquad (8)$$

$$\boldsymbol{L}_{rs} = \boldsymbol{L}_{sr}^{T} = L_{ms} \begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{\partial}{3}\right) & \cos\left(\theta + \frac{}{3}\right) \\ \cos\left(\theta + \frac{\partial}{3}\right) & \cos\theta & \cos\left(\theta - \frac{}{3}\right) \\ \cos\left(\theta - \frac{\partial}{3}\right) & \cos\left(\theta + \frac{}{3}\right) & \cos\theta \end{bmatrix} \qquad (9)$$

$\boldsymbol{L}_{rs}$ As well as expressed in the formula are two different block matrix, and they are transposed to each other, and the size of the two parameters are related to the position of the rotor, the elements inside are constantly changing parameters, which is the fundamental reason for the whole system in a nonlinear state. $\boldsymbol{L}_{sr}$

2.2 Voltage equation

The voltage balance equation refers to the voltage value loaded on the three-phase stator windings of A, B and C, which can be expressed by the formula (10). $u_A = i_A R_s + \frac{d\psi_A}{dt}$ $u_B = i_B R_s + \frac{d\psi_B}{dt}$ $u_C = i_C R_s + \frac{d\psi_C}{dt}$

Corresponding to formula (10), all the voltage values loaded on the three-phase rotor windings of A, B and C can be converted to the stator side, and the resulting voltage equation can be expressed by formula (11) : $u_a = i_a R_s + \frac{d\psi_a}{dt}$ $u_b = i_b R_s + \frac{d\psi_b}{dt}$ $u_c = i_c R_s + \frac{d\psi_c}{dt}$

In formula (11), represents the value of the instantaneous change of phase voltage loaded on the stator and rotor; $u_A$、$u_B$ $u_C$ $u_a$ $u_b$ $u_c$ $R_s$、$R_r$ Represents the resistance value formed by the stator and rotor windings themselves.

All the voltage engineering is expressed in the form of a matrix, which can be expressed by the formula (12) :

$$\begin{bmatrix} u_A \\ u_B \\ u_C \\ u_a \\ u_b \\ u_c \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 & 0 & 0 \\ 0 & 0 & R_s & 0 & 0 & 0 \\ 0 & 0 & 0 & R_r & 0 & 0 \\ 0 & 0 & 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & 0 & 0 & R_r \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \psi_A \\ \psi_B \\ \psi_C \\ \psi_a \\ \psi_b \\ \psi_c \end{bmatrix}$$

Or written as (12-a) $\boldsymbol{u} = \boldsymbol{Ri} + \frac{d\boldsymbol{\theta}}{dt}$

By combining the previously derived flux equation with the voltage equation, a new voltage equation can be obtained, which can be expressed by formula (13) :

$$\boldsymbol{u} = \boldsymbol{Ri} + \frac{d}{dt}(\boldsymbol{Li}) = \boldsymbol{Ri} + \frac{d\boldsymbol{i}}{dt} + \frac{d\boldsymbol{L}}{dt}\boldsymbol{i} = \boldsymbol{Ri} + \boldsymbol{L}\frac{d\boldsymbol{i}}{dt} + \frac{d\boldsymbol{L}}{d\theta}\omega\boldsymbol{i} \quad (13)$$

In formula (13), the value of electromotive force is also changing because the current value flowing through the system is constantly changing. This value is called pulse electromotive force (or transformer electromotive force); $\boldsymbol{L}\frac{di}{dt}$ $\frac{d\boldsymbol{L}}{d\theta}\omega\boldsymbol{i}$ Represents because the stator and rotor phase winding between the position in a constantly changing state, resulting in a proportional relationship with the speed of the electromotive force value is also constantly changing, this parameter is called rotational electromotive force.

## 3 Conclusions

This paper takes asynchronous motor as the research object, analyzes the mathematical model of asynchronous motor itself, and obtains the related

Flux equation, voltage equation, torque equation and motion equation, at the same time, the model can fully reflect the speed regulation system of three-phase asynchronous motor, even if the current and voltage harmonics in the circuit are relatively large. It lays a good foundation for the following research. But the research here is only limited to the theoretical level, the next step needs to use simulation software or experimental platform to achieve.

## References

[1] Huibin Ma. Development status and trend of new energy vehicle technology [J]. Science and Technology Economic Market,2016,(04):12.

[2] Xiaoxu Zhou. Design of AC asynchronous Motor Variable frequency Speed Control System [J]. Journal of Anhui Electronic and Information Vocational and Technical College,2017,(01):90-94.

[3] Jing Liu. Design of Three Phase Asynchronous Motor with Variable frequency Speed Regulation [J]. Electric Machine Technology,2017,(04):5-9.

[4] Xiongyou Qiu, Lin Lin,Hui Li. Design of Variable frequency speed Regulation System for asynchronous Motor [J]. Journal of Shaoyang University (Natural Science Edition),2017,(02):27-31.

[5] Xuelin Yang. Characteristics and Variable Frequency Speed Regulation Analysis of three-phase Asynchronous Motor [J]. Industrial Technical Innovation,2016,(01):69-73.

[6] Minghao Yang. Discussion on Characteristics and Application of Variable frequency speed Control asynchronous Motor [J]. China High-Tech Enterprises,2016,(03):55-56.

[7] Junliang Lai,Jiawen Song. Discussion on Design and Manufacturing Technology of Variable Frequency Speed Regulating Asynchronous Motor [J]. China New Technology and New Products,2015,(12):29-29.

[8] Yi Ruan, Xiaohua Zhang. Magnetic Field Orientation Model and Control Strategy of Asynchronous Motor [J]. Electric Drive,2002,(3):3-5.

**About the Author:**

Yan Lu (1990-), female, born in Weinan, Shaanxi Province, lecturer. Her main research fields are power electronics and power transmission:

Guangwei Cheng (1991-), male, born in Heze, Shandong Province, lecturer, mainly engaged in underground and tunnel engineering technology