# Downstream Entry in a Supply Chain

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Among manufacturing consumer goods industries, many firms compete in the upstream of the supply chain, however, scant attention has been paid to the effect of downstream entry in a two-tier supply chain (manufacturers in the upstream and retailers in the downstream for simplicity) with multiple manufacturers competing in the upstream. This paper extends Tyagi's model and introduces $m$ and $n$ firms that compete in quantity in the upstream and downstream market, respectively. In special cases, when $m$ equals one or approaches infinity, our model will reduce to Tyagi's or Frank's model, respectively. Therefore, our model can be deemed as an integrated model of entry issues.

**KEYWORDS:** the supply chain ; two-tier supply chain; Downstream Entry

## 1. THE MODEL

### 1.1 MODEL NOTATIONS

To keep our analysis simple and clear, we introduce notations with their meaning shown below.

| Notation | Explanation |
|---|---|
| $n$ | The number of retailers in the downstream, $n>1$ |
| $m$ | The number of manufacturers in the upstream, $m>0$ |
| $q_2^j(\theta_j)$ | The output of manufacturer $j$ and $\theta_j$ might be some random variable that influences manufacturer $j$ 's individual output |
| $q_1^i$ | The output of retailer $i$ |
| $q_1$ | The individual output of each downstream incumbent in equilibrium due to symmetry |
| $Q=\sum_{i=1}^{n} q_1^i=\sum_{j=1}^{m} q_2^j$ | The total output of the whole channel |
| $w(n,Q)$ | Wholesale price for each manufacturer |
| $w'$ | $w'=\frac{\partial w}{\partial Q}$, we use the two interchangeably |
| $p(Q)$ | Retail price for each retailer |
| $\Pi_i$ | The profit of retailer $i$ |
| $\Pi$ | The profit of each downstream incumbent in equilibrium due to symmetry |
| $\Psi_j$ | The profit of manufacturers $j$ |
| $\sum\Psi$ | The total profit of manufacturers |
| $\varphi$ | The elasticity of slope of the inverse consumer demand |

### 1.2 MODEL

Before presenting our model, we discuss the key aspects of our analysis and introduce the general game framework. We consider a two-tier supply chain in which firms compete at each tier. For convenience we refer to the upstream channel members as manufacturers and the downstream firms as retailers. Our model rests upon the following assumptions.

1.We assume all of $n$ retailers are identical and accordingly the number of retailers, $n$ , is endogenous in the model as Seade (1980a) has noted that effects of entry may be examined in an oligopolistic market through the number of firms as a parameter in the model. Furthermore, we assume $n$ to be larger than one ( $n>1$ ) which means that there should at least be one incumbent and then entry increases the number of retailers by one. Or less, it is not clear what we mean by "the downstream incumbent(s)". With this assumption, effects of downstream entry are exclusively driven by competition among retailers and between two tiers.

2.We assume the post-entry game, which is simultaneous moving and static, between retailers and is Cournot-Nash; that is, each retailer- having observed the wholesale price- now selects its output in order to maximize its own profits mutually, taking as given the output of the other firms. The assumption of Cournot competition in a homogeneous product market is ubiquitous (Frank 1965, Seade 1980(b), Jeuland and Shugan 1983, Tyagi 1999, Corbett and Karmarkar 2001). Or else, based on Bertrand's theorem, if more than one firm produce (at zero costs) a homogeneous product and compete in prices without cooperation, then the only Nash equilibrium is one where firms implement marginal cost pricing, thus reaping zero profits. For the same reason, we assume $m$ manufacturers also compete in quantity. Cournot competition is only a sufficient rather than necessary condition to arrive at our result and, moreover, a quasi-Cournot downstream market which allows some degree of Collusion is already sufficient, see Seade (1980b). Our main conclusion does not hinge on the assumption of symmetry among (between) manufacturers in our study.

3.We assume manufacturers and retailers face the inverse retailer and customer demand curves as $w(Q)$ and $p(Q)$, respectively.

$$\frac{dw}{dQ} = w^{'} < 0 \tag{3-1}$$

$$\frac{dp}{dQ} = p^{'} < 0 \tag{3-2}$$

(3-1) and (3-2) guarantee the inverse demand curves with a downward slope which accords with the traditional economic theory. Later we will show that the assumption of $w^{'} < 0$ is actually the same as stability condition.

4.In this game, manufacturers and retailers behave as the Stackelberg leader and follower, respectively.

5.We assume to produce one unit of the final product, retailers require the same number of inputs from manufacturers and, without loss of generality, the number is set to one.

6.We assume that to produce one unit of the final product, retailers face a flat marginal cost; that is, the only cost retailers bear is the wholesale price and the processing cost for retailers is normalized to zero since the processing cost will not influence the result given homogeneity among retailers.

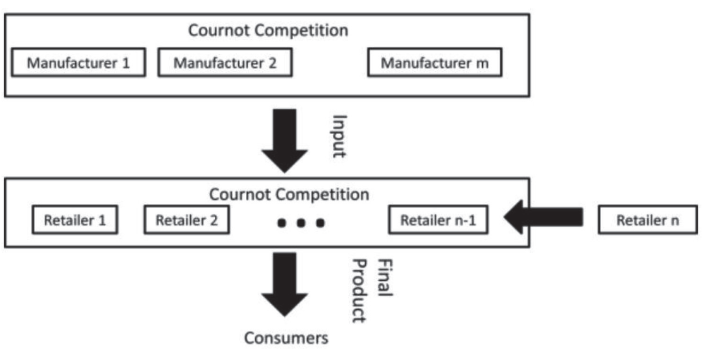


**Figure 1. Game Structure of the Supply Chain**

Figure 1 shows the basic structure of this manufacturer-retailer supply chain. In the upstream, manufacturers compete in quantity with each other and given the inverse demand function, $w(Q)$, mutually set a wholesale price $w$. In the downstream, $n$ retailers also follows Cournot competition and given the whole sale price $w$ and the inverse demand function, $p(Q)$, decide on the retailer price $p$.

We first study post-entry competition. Clearly, the cost for each retailer to produce one unit of products is the wholesale price determined in the upstream. Retailer $i$ will choose its own output to maximize its own profit and maximize its profit function,

$$\max_{q_1^i} \Pi_i = q_1^i(p(Q) - w(n,Q)) \tag{3-3}$$

To find the optimal output for each retailer, we taking the first order and second order condition of (3-1) w.r.t. $q_1^i$,

$$p(Q) - w(n,Q) + q_1^i p^{'} = 0 \tag{3-4}$$

$$2p^{'} + q_1 p^{''} < 0 \tag{3-5}$$

Since retailers are symmetric, they will choose the same output $q_i$. So, we get

$$p(Q) - w(n,Q) + q_1 p^{'} = 0 \tag{3-6}$$

$$2p^{'} + q_1 p^{''} < 0 \tag{3-7}$$

Whereas, to make the downstream Cournor market stable, we need further requirement1,

$$(n+1)p^{'} + nq_1 p^{''} < 0 \tag{3-8}$$

(3-8) states the condition of establishing a stable Cournot-Nash Equilibrium in the channel. Let the elasticity of slope of the inverse consumer demand function be $\varphi = \frac{nq_1 p''}{p'}$ and we can rewrite (3-8) into

$$\varphi > -(n+1) \quad (3\text{-}9)$$

The stability is important for our research for several reasons. First, it is noted that the symmetric equilibrium is unstable in the sense of not being the limit of Cournot dynamics; that is, if the equilibrium we reach is not stable, a slight deviation in output by one retailer will cause divergences of all other retailers away from the equilibrium, and then the equilibrium cannot be restored in practice. For example, if retailers find it profitable to increase (decrease) its own output when any one of retailers did so, the initial equilibrium is upset. Second, as Dixit(1986) has put it, in oligopoly theory we are focused on the comparative statics of the equilibrium and stability condition help fix many signs of variables in comparative statics. Third, stability condition is the necessary and sufficient condition for Nash equilibria of static games to be observationally equivalent to singe optimization problems, see Slade(1994). Last, we will show later that stability condition is equivalent to assuming that the slope of inverse retailer demand function is smaller than zero in our Cournot-Cournot setting. Accordingly in the model for retailers competing in quantity, we need to make sure that the downstream market is in stable condition which requires that the marginal profitability of each retailer decreases (increases) with an increase (decrease) in total retail outputs.

From the implicit function (3-8) of $q_1$, by using partial differential equation, we can get

$$\frac{\partial q_1}{\partial n} = \frac{-q_1\left(p' + q_1 p''\right)}{\left(n+1\right)p' + nq_1 p''} \quad (3\text{-}10)$$

$$\frac{\partial q_1}{\partial w} = \frac{1}{\left(n+1\right)p' + nq_1 p''} \quad (3\text{-}11)$$

From stability condition, we know that the denominator of (3-10) and (3-11) is negative. It is obvious that the sign of $\frac{\partial q_1}{\partial n}$ is uncertain without knowing the sign of $p' + q_i p''$. However, we find that $\frac{\partial q_1}{\partial w}$ is definitely negative which means that each retailer will respond to the increase (decrease) of the wholesale price by reducing (raising) its individual output.

Rearranging (3-6), we can get the inverse demand function for the upstream market,

$$w\left(n,Q\right) = p\left(Q\right) + p'q_1, \quad (3\text{-}12)$$

which is equivalent to

$$w\left(n,Q\right) = p\left(Q\right) + \frac{1}{n}Qp'. \quad (3\text{-}13)$$

This (3-13) shows that the inverse retailer demand function, $w\left(n,Q\right)$, is a function of the total output, $Q$, and the number of retailers, $n$. It means that downstream entry affects the wholesale price through both $n$ and $Q$.

In the upstream, manufacturer $j$ maximizes its payoff,

$$\max_{q_2^j(\theta_j)} \Psi_j = w\left(n,Q\right) \times q_2^j\left(\theta_j\right). \quad (3\text{-}14)$$

$q_2^j\left(\theta_j\right)$ means that manufacturer $j$ 's individual output might be contingent on some random variable $\theta_j$ and therefore manufacturers are not symmetric. We do not need the assumption of manufacturer symmetry to derive our results. We just need to remember that manufacturers are not necessarily symmetric and we will omit $\theta_j$ later on for consistency.

The first and second order condition of $\Psi_j$ w.r.t. $q_2^j$ is as follows.

$$w\left(n,Q\right) + w'q_2^j = 0 \quad (3\text{-}15)$$

$$2w' + w''q_2^j < 0 \quad (3\text{-}16)$$

Based on the stability condition of the upstream market, we have

$$\left(m+1\right)w' + Qw'' < 0 \quad (3\text{-}17)$$

Summing the first order condition of all manufacturers, we get

$$mw\left(n,Q\right) + w'\sum_{j=1}^{m} q_2^j = 0 \quad (3\text{-}18)$$

Since $\sum_{j=1}^{m} q_2^j = Q = nq_1$, we can rewrite (3-17) as

$$mw\left(n,Q\right) + nq_1 w' = 0 \quad (3\text{-}19)$$

Next, we will study effects of entry on the total output in Section 3.3. Then we will examine how entry affects the individual output and profitability of retailers.

## 2. CONCLUSION

In this paper, we examine entry problem with a model with multiple manufacturers and multiple retailers. Both manufacturers and retailers compete in Cournot equilibrium in their each level. Entry happens when a new retailer that are homogeneous with other downstream incumbents. One of the implications is that under suitable number of retailers and market structures in both downstream and upstream and incumbents in downstream should encourage entry because it is profitable.

Future research can be conducted in the following aspects:

(1)Model in this paper does not examine what will happen when some or all downstream incumbents collude. It is an interesting topic about the influence of collusion on entry decision.

(2)This paper only assumes linear demand function on both levels in the supply chain. Other more sophisticated demand function can be test to see whether we can get similar result.

## REFERENCE

[1] Amir, R., V.E. Lambson. 2000. On the Effects of Entry in Cournot Markets. Review of Economic Studies. 67 235-254.

[2] Arya, A., B. Mittendorf, D.E.M. Sappington. 2007. The Bright Side of Supplier Encroachment. Marketing Science. 26 651-659

[3] Bresnahan, T.F. 1981. Duopoly Models with Consistent Conjectures. The American Economic Review. 71 934-945.

[4] Choi, S. C. 1991. Price competition in a channel structure with a common retailer. Marketing Sci. 10 271–296.

[5] Cournot, A. 1927. Research into the Mathematical Principles on the Theory of Wealth. The Macmillan Company, New York, NY.