Simplifying the complex and abstract concept of calculus -- Taking the concept of limit as an example

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Abstract: calculus is a very important content of higher mathematics, and it is also a compulsory course of basic economic mathematics for some economics and management majors. However, the concept of calculus is very abstract, and there are many symbols in the concept, so the concept of calculus has always been a difficulty in teaching and students' understanding. Therefore, when most students learn calculus, they begin to have irresistible resistance from the concept, which leads to the gradual decline of their enthusiasm for calculus learning. Therefore, the teaching of calculus abstract concept is very important. We should try to instantiate the abstract concept, integrate theory with practice, make the abstract concept easy to understand, and pave the way for the subsequent study of calculus content, so that students can experience the wonder and fun of mathematics in their future study.

Key words: calculus; Abstract concept of calculus; Instantiation of abstract concepts; Integrating theory with practice

1 The origin of calculus

Calculus is a branch of mathematics that studies the differentiation and integration of functions and related concepts and applications. It is a basic subject of mathematics. Calculus was born in the 17th century, mainly from the huge impetus of politics, economy and society to mathematics. At that time, mathematicians could not solve the following problems: finding area, volume and acceleration; In solving these practical problems, a series of explorations and researches were gradually produced, such as solving tangent problems and extreme value problems. Then in the second half of the 17th century, calculus was initially formed through the improvement of Newton and Leibniz, and it was not until the 19th century that the theory of calculus was improved and used today.

The limit thought in calculus, as early as the 3rd century A.D., has been produced in ancient China. At that time, when the famous mathematician Liu Hui calculated the PI, he created the circle cutting technique, which is the typical application of limit thought. Liu Hui pointed out that the circumference of a circle consists of multiple positive 6×2N-1 is composed of the perimeter of the side shape. With the constant increase of the coefficient n, this value is also infinitely close. To verify this conclusion, he set the circumference of the circle as s and inscribed the circle with 6×2N-1 regular polygon is set as Si (i=1,2, n). With the constant increase of coefficient n, S1, S2,... Sn form a sequence of numbers. When the value of n is larger, the error between the circumference of the inscribed regular polygon and the

[7] Kai Liu,Jing Zhou Research on mixed user equilibrium model under traffic information guidance [J]System engineering theory and practice, 2020, 40 (2): 415-425

[8] Shuanglin Li,Zujun Ma Disaster relief route selection based on user equilibrium condition under post-earthquake traffic control [J]Journal of management engineering, 2014, 028 (003): 148-155217

[9] Xinhua He,Wenfa Hu,Xizhao Zhou, etc Collaborative optimization model of post-earthquake emergency supply problem and traffic network [J]China management science, 2017, 25 (04): 104-114

[10] Haitao Chen,Feng Bai,Mingmin Fang Research on communication mechanism of urban emergency management based on multi agent [J]Information science, 2010, 28 (12): 1884-1888

[11] Zifu Fan,Lihua Wang,Hong Sun Construction of emergency communication classification model based on dominance rough set [J]Statistics and decision making, 2015, 18 (4): 56-59

[12] Xing Chen,Yong Wang,Lingyun Wu, etc Multi stage, multi-objective and multi department emergency decision-making model [J]System engineering theory and practice, 2010, 30 (11): 1977-1985

[13] Xinhua Mao,Jianwei Wang,Changwei Yuan, etc Research on post disaster highway network repair scheduling based on elastic optimization [J/OL] Chinese Journal of highway: 1-18
http://kns.cnki.net/kcms/detail/61.1313.U.20210309.1404.002.html, 2022-04-26

[14] Mengling Zhang,Jing Wang,Jun Huang, etc Research on Integrated Optimization of road repair and material distribution under emergencies based on mobile positioning data [J]China management science, 2021, 29 (3): 133-142

[15] Binghua Tu Research on emergency repair team scheduling problem for severely damaged road network [D]Hefei University of technology, 2020

circumference of the circle is smaller. However, regardless of the value of coefficient n, the value of Sn is the circumference of the inscribed regular polygon, which is also the embryonic form of the idea of "limit". At the same time, the enlightenment brought by the above practice is: connect the regular polygon 6 in a known (such as a circle×There is still a difference between the approximate accuracy of 2N-1 (SN) and the unknown (such as the circumference of a circle s). As long as they are equal under a special condition, and can achieve a leap to qualitative change through unlimited quantitative change, this scientific idea is also the core connotation of the concept of calculus.

Differential calculus, including the operation of derivative, is a set of theories about the rate of change. It makes the function, velocity, acceleration and slope of the curve can be discussed with a set of general symbols. Integral calculus, including the operation of integral, provides a set of general methods for defining and calculating area and volume.

2 The significance of calculus

The creation of calculus was not accidental. It was entirely created by Cui Sheng, who was in need of social development and scientific and technological development at that time. By the 17th century, the creation of calculus had brought great impetus to the development of mathematics. In the past, many problems that could not be solved by elementary mathematics, such as calculating the area of surface graphics and the volume of surface geometry, were solved by calculus, All these problems can be solved easily, which is enough to show the charm of calculus. The position of calculus in the development of mathematics is very important. It can be said that it is the largest creation in all mathematics after Euclidean geometry. Its contribution to the development of human society is immeasurable. Until now, calculus plays a vital role in many fields and makes a great contribution to the development of modern economy and society.

3 Teaching difficulties of calculus

Calculus symbols are numerous and complex, the definition is abstract and difficult to understand, and it is difficult to contact with practice, which has always been the difficulty of teaching. For example, starting from the concept of limit, many students do not really understand the concept of limit and are not very clear about the symbol of limit. Most students think that they can calculate the limit by applying formulas and rules. This kind of mechanical learning method showed its disadvantages when learning derivative in Chapter 2, because the concept of derivative and the concept of limit are related. If the limit is not understood, then the concept of derivative can not be understood, and then a chain reaction occurred. Because the concept of differential is related to the concept of derivative, then the concept of differential behind is still not understood, Over time, there are more and more ambiguous contents, and students will gradually become resistant to calculus, and even completely give up mathematics. Therefore, at the beginning, the teaching of calculus concepts must be paid attention to, and the concepts must be stripped and cocooned. The meaning of each symbol is clearly explained, so that students can connect concepts with practical applications, so as to truly apply what they have learned, Promote professional learning.

4 Methods that should be paid attention to in the process of extreme learning

According to the distribution of calculus knowledge points, teachers should first clarify the main reasons why this knowledge is difficult to master. The so-called limit conceptε-δDefinition is actually the interrelation and reflection between results and processes. This process can be analyzed from a dynamic perspective or characterized by static finite quantities. At the same time, theε-δThe definition can fully reflect the dialectical relationship between the finite, infinite potential, potential infinite and real infinite of functions. This logical relationship is the focus of teachers' teaching work, which is also the difficulty for students to master knowledge. In this regard, in the process of teaching, teachers should analyze both students' and teaching education

First, students' learning. The concept of limit is a dialectical concept. Before learning, students should have the corresponding level of thinking logic to accurately understand the key and difficult knowledge such as definitions and formulas. From their own point of view, students should have the abilities of intuitive action thinking, concrete action thinking, logical formal thinking and so on. At the same time, according to the development of thinking, students should also have good thinking dialectical ability and logical reasoning ability. In the form, logic is the main feature of learning, that is, certainty and abstraction, so as to better understand this abstract knowledge.

Secondly, teachers' teaching. Before carrying out extreme teaching, teachers should deeply understand the characteristics and connotation of the idea of limit, reasonably infiltrate it into the mathematics classroom around the teaching content and curriculum standards, change the previous "knowledge led" teaching mode, strengthen the development and application of teaching resources and teaching services, simplify students' understanding, and make them learn by doing while learning. In actual teaching, teachers should first eliminate students' fear of difficulties, combine the content of limit with daily life, "simulation" and "slow motion" analysis, so that students can fully understand how to raise questions and what direction to start research, so as to improve their learning enthusiasm.

5 Instantiate the abstract concept of calculus to stimulate students' interest in learning and promote the effect of classroom teaching

The first chapter of general calculus content is function, but the content of function is basically the same as that of middle school function, so it will not be introduced here. The first section of the second chapter deals with the basic content of calculus - the concept of limit. Generally, the definition of sequence limit is given first, and then the definition of function limit is given. Most of the limit concepts are defined as follows: set sequence$\{x_n\}$, if a constant exists$a$, for any given positive number$e$, there is always a positive integer$N$, properly$n>N$Time, inequality$|x_n-a|<e$Is true, then it is called constant$a$Is a sequence$\{x_n\}$The limit of, or sequence of numbers$\{x_n\}$

Convergence to$a$, recorded as $\lim_{n\to\infty} x_n = a$. According to this definition, it is difficult for beginners to realize that limit describes the ultimate trend of the sequence of numbers from this definition. Firstly, the more mathematical symbols in the definition, the more difficulty it will be visually increased. The meaning of each symbol can not be intuitively felt. Secondly, the concept of limit does not directly reflect the trend of variables, but uses inequality$|x_n-a|<e$It is difficult for students to understand from this inequality$e$It is an infinitely small positive number and can be infinitely close to 0. Therefore, when learning the concept of limit, it is advisable to list some practical cases related to limit, and express the quantities represented by each symbol in the concept with some simple and understandable or common variables. This can not only improve students' interest in learning, stimulate students' spirit of exploration, but also enable students to have a profound and thorough understanding of the concept, Lay a good foundation for subsequent study. For example, case 1: in the process of climbing, our height is gradually rising and is infinitely close to the height of the top of the mountain. If we regard our height as a variable$h$As time goes on, hThe trend of change is that it is getting larger and infinitely close to the height of the top of the mountain. At present, the highest mountain on the earth is Mount Everest. That is to say, the limit of human height rise is Mount Everest's height of 8848.86 meters. If the heights of all the mountains on the earth are arranged from low to high, they form a series$\{x_n\}$Then, this sequence is an increasing sequence, and the sequence is ultimately infinitely close to and equal to 8848.86 meters, that is, it meets the inequality$|x_n-884886|<e$It is necessary to(eIs any real number greater than 0); Case 2: Zhuangzi Tianxia said, "the hammer of a foot is half taken every day and will never be exhausted for all ages". If the hammer of a foot is regarded as unit 1, then this sentence is translated in mathematical language as: take it on the first day $\frac{1}{2}$ The next day$\left(\frac{1}{2}\right)^2$ On the third day$\left(\frac{1}{2}\right)^3$, Take on the nth day$\left(\frac{1}{2}\right)^n$ Obviously, with the increase of time, the amount will be less and less, but it will never be completed, that is to say, it will never be exhausted, because$\left(\frac{1}{2}\right)^n$Although it is smaller and smaller, it is always greater than 0. If the amount taken every day is arranged into a sequence$\{x_n\}$, then this sequence is a decreasing sequence, and the sequence is ultimately infinitely close to 0, that is, it satisfies the inequality$|x_n-a|<e$It is necessary to(eIs any real number greater than 0); Case 3: cancer cells of patients with advanced cancer generally spread very fast in a short time. If the number of cancer cells proliferated every day is regarded as a series$\{x_n\}$Obviously, the general term of this sequence will always be larger and larger, but there is no upper limit, that is, there is no constant$a$Make inequality$|x_n-a|<e$This is also the reason why it is difficult to cure advanced cancer. In the above cases, the variables of the first two cases have a similar trend, that is, the variables are ultimately infinitely close to (tend to) a certain constant. Such a sequence of numbers is called a convergent sequence of numbers, and this certain constant is called the limit of the sequence of numbers. Obviously, this constant is in the definition of limit$a$ That is, the limit of the variable, and the inequality in the definition$|x_n-a|<e$It means that the variable is infinitely close to a constant$a$ This change trend. Obviously, the variation trend of the variables in case 3 is different from that of the first two. The variables ultimately do not tend to any certain constant. Such a sequence is called a divergent sequence, and it has no limit. After the actual case is given, the meaning of each symbol in the definition is elaborated in detail combined with the definition of sequence limit, so that students can thoroughly understand the definition of limit. Integrating theory with practice makes students feel that mathematics is still closely related and inseparable with us in life. The actual case is easy to understand, which can also arouse students' interest in learning, promote the effect of classroom teaching, and then pave the way for subsequent learning, making the following content easier to understand, and win a good start for mathematics learning.

6 Relying on mind map to strengthen students' mathematical thinking and improve students' learning efficiency

Compared with other mathematical knowledge, the knowledge of calculus is trivial and requires students to have good logical thinking and reasoning ability. From the perspective of the discipline itself, calculus is a rigorous course, which is composed of "definitions, formulas, methods, theorems, examples", etc. if students only use textbooks, it is difficult to understand. The core idea behind the knowledge of integration is also too dependent on textbooks, which can easily lead them to fall into a thinking stereotype and have a negative learning attitude. Therefore, in the process of teaching, teachers should get rid of the shackles of textbooks, strengthen the assistance of a variety of teaching tools, reduce the difficulty of students' understanding, and improve their learning enthusiasm and self-confidence. Among them, mind map shows good educational value. On the one hand, it can help students sort out the difficulties of chapter knowledge and clarify the main direction of learning; On the other hand, the combination of graphics can effectively reduce the difficulty of students' learning. With the help of images, it can form a good thinking framework, and it is easier to play its own thinking ability and learning ability according to the learning content. Taking the "lobitar's law in the differential calculus of functions of one variable" as an example, before teaching, teachers should deeply analyze the teaching objectives of this lesson, sort out the teaching difficulties involved, and draw the following mind map for students (as shown in Figure 1)

And let them study independently in the form of groups according to the thinking process, summarize the forms of Infinitives and the conversion relations of various types of indefinite time. In this process, teachers can also use multimedia technology to refine all the points of the guide map and show students the image of function change, so as to help them understand the limit forms of infinitives, Reduce students' rote copying and strengthen their mathematical thinking.

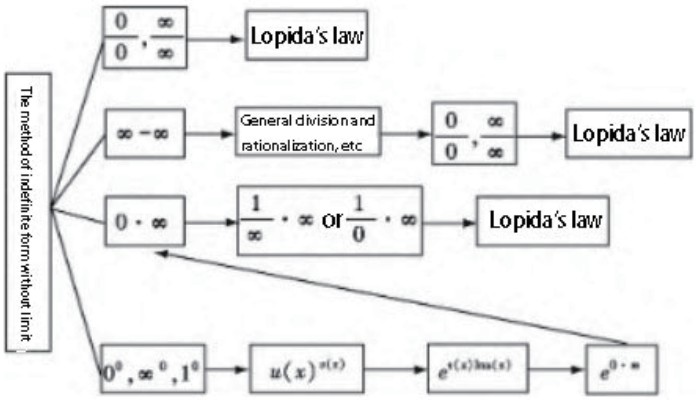

Fig. 1 mind map of indeterminate formula seeking limit

7 Conclusion

The thought and method of mathematical limit contains the thought and method of mathematical limit, and it is the dialectical unity of the thought and method of mathematical limit. At the same time, the concept of limit, as the basis of all concepts of calculus, is also an important premise for students to understand knowledge, apply knowledge and deepen their thinking. In order to prevent the abstraction of theoretical concepts from becoming a stumbling block for students' Learning Calculus, it is advisable to start with concepts in teaching, integrate concepts into actual cases, enrich classroom content, stimulate students' interest in learning, promote teaching effect, let students truly experience the mathematical thought of calculus, truly integrate theory with practice, improve students' practical application abilityAbility to solve problems comprehensively. In addition, in the course teaching, teachers should also pay attention to the discovery, proposal, infiltration, refinement, induction, generalization, summary, activation, deepening and application of mathematical thinking and methods, rely on a variety of teaching methods, improve the classroom temperature, improve the teaching form, so as to help students grasp the core idea of calculus and promote the comprehensive development of their own mathematical literacy.

References:

[1] Lianzhao Wang Attach importance to the teaching of calculus concept and improve students' application ability [g]Journal of Mathematical Medicine, 2016:12

[2] Ying Zhang,Xuchun Sun Analysis of calculus concept teaching methods in Higher Vocational Colleges [j]Research on vocational education, 2013:09

[3] Mingyu Chen Analysis of the teaching of the concept of calculus [j]China Science and technology information, 2010:10

[4] Yuxia Tang Discussion on the improvement and optimization path of calculus teaching in Higher Vocational Colleges [j]Mathematics learning and research, 2016 (17): 20

[5] Weijun Cheng Mathematics Culture: the only way to run through the effective teaching of calculus in Higher Vocational Colleges [j]Zhiyin Lizhi, 2016 (21): 196-197

[6] Min Xiao Analysis of innovation in Calculus Teaching Reform in Higher Vocational Colleges [j]Radio and TV University of science and technology, 2017 (04): 35-37

[7] Zhangcai Guan Exploration on the improvement and optimization path of calculus teaching in Higher Vocational Colleges [j]Mathematics learning and research, 2018 (05): 28

[8] Zhengyong Zhai,Guanglan Wang Application of calculus in Higher Mathematics in higher vocational teaching practice [j]Modern vocational education, 2018 (35): 140-141

[9] Yanlong Zhao Practical research on mathematical culture in Calculus Teaching in Higher Vocational Colleges [j]Reading abstracts, 2019 (12): 127

[10] Na Xue Introduction of mathematical culture in Calculus Teaching in Higher Vocational Colleges [j]Science lovers (education and teaching), 2019 (04): 26-27

[11] Daiqing Bai Practical exploration of mathematical culture in Calculus Teaching in Higher Vocational Colleges [j]Knowledge base, 2020 (04): 191

[12] Qing Wen Research on the teaching strategy of the concept of calculus of one variable function in Higher Vocational Colleges [j]Journal of Wuhan Institute of shipbuilding technology, 2020,19 (01): 55-57+66

[13] Xiufen Liu,Liqing Zhang Research on the theory and practice of infiltrating mathematical culture into mathematical calculus teaching in Higher Vocational Colleges [j]Mathematics learning and research, 2020 (18): 10-12

[14] Jiaojiao Luo Discussion on case teaching strategy based on calculus in Higher Vocational Colleges [j]Modern vocational education, 2020 (41): 118-119

[15] Youjun Zhou Ideological and political exploration and practice of calculus integration course in Higher Vocational Colleges [j]Journal of science and education, 2022 (06): 123-125+144