Analysis of continuity and derivability in Higher Mathematics Based on the cultivation of multidimensional innovation ability

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Abstract: the relationship between continuity and derivability in higher mathematics has always been a difficulty in the learning and teaching process of Higher Mathematics in Jiangsu Province. Because of the similarity of concepts, students have more confusion in the process of understanding. The comparative analysis between concepts is one of the effective methods to deepen the understanding of concepts. This paper analyzes the related concepts through an example of a topic, so as to help students deepen their understanding of the relationship between continuity and derivability and master relevant knowledge. At the same time, it explores the teaching methods to develop students' Multi-dimensional innovation ability.

Key words: Advanced Mathematics of "transfer from undergraduate to undergraduate"; Continuous; Differentiable; Continuous differentiable; Multidimensional innovation capability

Introduction

Advanced mathematics is one of the important subjects in Jiangsu "college to undergraduate" examination. In the process of teaching, it is found that the vast majority of students lack understanding of conceptual knowledge. The concepts of continuity, differentiability and continuous differentiability in differential calculus are typical and easily confused concepts. In the process of learning, students often mechanically memorize the conclusion that "differentiable must be continuous, but continuous is not necessarily differentiable". It is precisely because they do not deepen their understanding of the essence of the concept that students are confused when the words such as first-order continuous differentiable and second-order continuous differentiable appear in the proposition. The comparative analysis between concepts is an effective method to discriminate concepts. If many concepts can be discriminated through one question, it will get twice the result with half the effort for students' review. Through the discussion of many concepts such as piecewise function continuity and differentiability, this paper helps students strengthen their understanding of related concepts, improve their ability to analyze and solve conceptual problems, and try to explore teaching methods to develop students' multidimensional innovation ability.

1. differences of several concepts

There are several typical concepts of continuity and differentiability that need to be distinguished, namely, (1) continuity, (2) differentiability, (3) first-order continuous differentiability, (4) second-order differentiability, and (5) second-order continuous differentiability.

(1) Continuous $\lim_{x \to a} f(x) = f(a)$;

(2) Differentiable $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ Presence;

(3) First order continuous differentiable $\lim_{x \to a} f'(x) = f(a)$

(4) Second order differentiableExistence $f''(a) = \lim_{x \to a} \frac{f'(x) - f'(a)}{x - a}$;

(5) Second order continuous differentiable $\lim_{x \to a} f''(x) = f''(a)$.

So there are $(5) \Rightarrow (4) \Rightarrow (3) \Rightarrow (2) \Rightarrow (1)$ The direction of the arrow is unilateral, and reverse is not necessarily true. In the process of conceptual understanding, it is particularly necessary to distinguish between (2) and (3), (4) and (5). Take (2) and (3) for example, $f(x)$ Differentiable does not mean that $f'(x)$ Continuous, indicating only $f'(x)$ Exists, so if the known condition in the problem of finding the limit is $f(x)$Differentiable, even if the limit type is"$\frac{0}{0}$"Type or"$\frac{\infty}{\infty}$"Type, nor can we use the lobita rule, because it can not be guaranteed $f'(x)$ Continuous. Similarly, if the known conditions are The second-order derivative can be used only once. The last step is to use the derivative definition. It is necessary to It is necessary to It is necessary to It is necessary to It is necessary to

2. topic description

Assumed function $f(x)=\begin{cases} x^a \sin\frac{1}{x}, & x \neq 0 \\ 0, & x=0 \end{cases}$, Q:

(1)aWhen taking what range, $f(x)$stay Continuous?
(2)aWhen taking what range, $f(x)$ stay x=0 Place can guide?
(3)aWhen taking what range, $f(x)$ stay x=0 Where is continuous differentiable (i.e., derivative continuous)?

3. example analysis

It is not difficult to find from the observation of the above problems that it is inevitable to discuss whether it is continuous or differentiable or continuous differentiable x ® 0 When, $\sin\frac{1}{x}$ At this time $\sin\frac{1}{x}$ Finally, it tends to sin ¥ , although it cannot be accurately determined sin ¥ But sin ¥ Is essentially a [-1,1] Bounded quantity between. Thus we can immediately match an important test point of the limit part, that is, the bounded quantity of infinitesimal multiplication is still infinitesimal.

Although this topic does not further discuss the second-order derivable and second-order continuous derivatives, the idea of the whole problem is clear, and the later the requirements are more stringent. The continuity of derivatives can be discussed only when the existence of derivatives is ensured. Similarly, it is not difficult to understand that only when n Only when the first derivative is continuous can we further explore it n Order continuous differentiable. It is necessary to It is necessary to It is necessary to It is necessary to It is necessary to

4. extension and discussion

For the study of a course, especially the exam oriented course, it is important to reflect and summarize the problems in time. Regular "making a mountain out of a molehill" is very important for the consolidation and connection of knowledge. Mechanically carrying out the "sea topic tactics" will only achieve twice the result with half the effort, and eventually produce disgust for the course learning. In response to the unpredictable proposition techniques, we should extend our knowledge more, and the effect of making the topic "refined" is far more obvious than that of making a more "extensive" and "more" topic.

As far as this topic is concerned, it can be associated with sin D In this question x ® 0 When, is to $\sin\frac{1}{x}$ Performing limit operations as bounded variables. It is not difficult to find out from the analysis of the topic writing techniques over the years sin D There are only two kinds of thinking paths when D® 0 The method of processing is to sinD~D At this time, regardless of the specific form of the internal expression, just ensure that the internal D Such operations can be carried out when the whole is a quantity tending to 0; In addition, when D When it does not tend to 0, it is always regarded as a bounded variable, thinking that the bounded variable of infinitesimal multiplication is still infinitesimal, and it is also not concerned at this time D We only need to determine its trend. In other words, in the previous question, even if sin Not internally $\frac{1}{x}$ As long as it does not tend to zero, the final solution will not change.

With a little thought, the following variant problem can be derived: let function $f(x)=\begin{cases} x^a \sin x, x \text{ ¹ } 0 \\ 0 \quad, x=0 \end{cases}$, Q:

(1)a When taking what range, $f(x)$ stay x=0 Continuous?

(2)a When taking what range, $f(x)$ stay x=0 Place can guide?

(3)a When taking what range, $f(x)$ stay x=0 Where is continuous differentiable (i.e., derivative continuous)?

At this time, the expression has not changed much, but it is a completely different perspective x ® 0 When, sin x Instead of regarding it as a bounded quantity, we need to use sin$x$~x How to handle.

Such an analysis and discussion has sublimated such a simple case, and the original mundane topic instantly emanated from multiple perspectives. Further extension and discussion also expand the students' perspective of thinking about problems, and such exploration should be carried out in the future teaching practice.

For another example, the definition of derivative is more abstract, and students often have some understanding problems, which affect their subsequent understanding and application of this concept. Because elementary mathematics is often "static", students lack a more intuitive understanding of "infinite approach" in limits. For example, when explaining the concept of derivative, most of the textbooks use "instantaneous speed" to express, so that students can calculate the average speed of a certain period of time relatively quickly, but it is difficult to understand the concept of "instantaneous speed" at a certain point, and teachers need to guide students step by step.

First, assume that∆The object moves in the time of T, calculate∆The average speed of the object in time t is∆V=∆S/∆t. When∆T tends to infinite hours, $\Delta V \approx \bar{v}$, but no matter∆How t approaches infinitesimal, ∆V is infinitely close to the instantaneous velocity, that is to say, it is only an approximation of the instantaneous velocity, only when∆When t → 0, when the limit value exists, we think that the limit value is the instantaneous velocity at this time. After students understand the algorithm of instantaneous velocity, they can abstract the concept of derivative from it. Students can understand that derivative refers to the instantaneous rate of change of a function at a certain point. After students understand the concept of derivative, the teacher can use several examples to deepen students' understanding of the concept of derivative. For example, marginal cost, marginal income and marginal profit in economics are the limits of the ratio of economic variables to independent variables. It is an important analytical method in economic theory to study the changes of economic variables through derivatives.

By introducing the "instantaneous velocity" in physics, students can analyze the concept of derivative from the perspective of different disciplines. At the same time, students can use the old concept to lead to new concepts, so that students can have a more intuitive understanding of the concept of derivative. In addition, students can further understand that the problem of the rate of change of function can be solved through derivative, Deepen students' understanding of the concept of derivative, let students master this knowledge point, apply this knowledge point of derivative to practice, realize the transfer of mathematical thinking, and develop creative thinking.

5. conclusion

There is an efficient method in scientific thinking, that is, all research should start from simplicity, understand complex concepts from simple examples, and find extraordinary perspectives and changes from simple problems. This is innovation. This kind of innovation in teaching can not only improve students' problem-solving ability and concept understanding, but also promote students' flexibility of thinking, which is also conducive to cultivating students' further innovation in future practical work. This paper is not only a simple example analysis of the specific knowledge point of continuity and derivability in Jiangsu "college to undergraduate" higher mathematics, but also an attempt to explore the new "college to undergraduate" higher mathematics teaching method.

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