Two Dimensions of Financial Asset Pricing: Expected Cash Flow and no Arbitrage

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Abstract: In terms of intrinsic value measurement, different from general commodities, financial assets pursue the right to claim future returns, and their value is reflected in the expected rate of return, which is determined by the expected cash flow brought by the unit asset. In the market supply and demand equilibrium, the price of general commodities is determined by the traditional economic supply and demand equilibrium, while the value of financial assets is determined by the arbitrage equilibrium. This paper presents a mathematical model of the return on financial assets in two periods, and explains how to estimate it from the perspective of expected cash flow and no arbitrage.
Keywords: Asset Pricing; Cash Flow; No Arbitrage

1. Introduction

In the case of general commodities, their real value is determined by their utility. According to the economic value theory of commodities, the value of commodities depends on their marginal utility. If the consumption budget of household income and the indifference curve of demand goods are placed in a coordinate system, the equilibrium point of household consumption utility maximization is obtained. In the optimal equilibrium state, the ratio of commodity price is equal to the ratio of commodity marginal utility, and the relationship between commodity price and commodity value is established. For financial assets, it is the right to claim future income, that is, to buy and hold a financial asset is to obtain the ownership of the future income cash flow of this financial asset. Therefore, the value of financial assets depends on future earnings cash flows. Like general commodities, if financial assets are also regarded as a commodity, the value of financial assets also depends on marginal utility, and the ratio of two financial assets' prices is equal to the ratio of their marginal utility, while the price of financial assets is the future expected rate of return, that is, the future income cash flow per unit of financial assets.

Compared with the market supply and demand equilibrium of ordinary commodities, the market transaction equilibrium of financial assets has its particularity, which is based on the particularity of financial assets different from ordinary commodities. First of all, in a perfectly competitive financial market, the expected rate of return of financial assets with the same risk will be the same, otherwise, market participants will sell financial assets with low rate of return and buy financial assets with high rate of return to carry out arbitrage. Arbitrage behavior will make the expected rate of return of all financial assets with the same risk in the market eventually converge, and then establish arbitrage-free equilibrium. Second, financial assets establish equilibrium more quickly than commodities in general. In the general commodity market, once the price is out of equilibrium, most suppliers and demanders will take action, but each supplier and demander will only adjust their supply and demand in a small amount, the market will gather their adjustment, will produce a large supply and demand adjustment, thus promoting the price reset. However, arbitrage in the financial market is not the same, once the arbitrage opportunity is found, only a few arbitrageurs (in theory, even only one) can use the short mechanism to establish a huge arbitrage position to promote the correction of the unbalanced price. Therefore, the supply and demand pressure generated by arbitrage is very large, and the speed of rebuilding the equilibrium is much higher than the adjustment of the supply and demand gap in the general commodity market.

It is precisely the difference in the establishment mechanism of market equilibrium between financial assets and general commodities that their pricing methods are also different. The price of general commodities is determined by supply and demand equilibrium, while the price of financial assets is determined by arbitrage equilibrium.

2. The mathematical model of the return on financial assets

The rate of return of different financial assets has different characteristics. For example, Treasury bills, which are zero-coupon securities, have a relatively short term, up to one year, the yield is equal to zero, and the purchase price and maturity price are unchanged. Therefore, there is no rate of return on assets, only the rate of appreciation, and the rate of appreciation is fixed, so the rate of return on assets is also fixed. On the other hand, stock assets generally have uncertain dividends, and the purchase price and the ending price will change. Therefore, the return rate and value-added rate of stock assets are random variables, and so is the return on assets.

Whether certain or uncertain, asset returns and future asset prices are future values, and the initial value of the present moment, like the historical value, is certain, that is, once determined, it will not change.The relationship between rate of return $r(T)$, yield $r_{yield}(T)$ and value-added $r_{value}(T)$ rate is as follows：

$$r(T) = r_{yield}(T) + r_{value}(T)$$

As long as either the return on assets or the value added rate is random, the return on assets is random.

If PV represents the present value, FV represents the final value, $V(t_0)$ represents the initial value of the asset, $V(t_0 + T)$ represents the value after time T , and $Y(T)$ represents the return of the asset in T . Due to the following relationship：

$$PV = V(t_0)$$

$$FV = Y(T) + V(t_0 + T)$$

$$PV \cdot [1 + r(T)] = FV$$

The above relationship is simplified to the relationship between present value and final value, and knowing the value of any two variables can determine the value of the third variable. But in reality, we often face such a choice: according to the family's risk appetite, combined with the current expected cost of its own money capital (in determining the cost, the investment period is determined), choose an asset that is close to its expected rate of return, then at what price is closer to the value of the asset? The process of solving this choice problem is the process of asset pricing. Therefore, the basic equation of asset pricing is:

$$PV = \frac{FV}{1 + r(T)}$$

If interest is continuously compounded during the investment period, the basic equation of asset pricing according to the continuous compounding formula is:

$$PV = FV \cdot e^{-r(T)}$$

The above process is essentially a kind of discount, and the final value can be regarded as the future cash flow. However, in practice, especially high-yield assets, the final value is difficult to determine, are all random variables, showing different random distributions.

3. Estimated expected cash flow of financial asset value

The pricing of financial assets is to find the value of PV , while $Y(T)$ and $V(t_0+T)$ are both future values and future cash flows brought by financial assets. In particular, it is necessary to participate in the estimation in the form of cash flows, rather than some undefined equity.

Both $Y(T)$ and $V(t_0+T)$ are uncertain variables that can be either independent or correlated; It can be a single asset or a combination of multiple assets. However, it can be classified as the following problem: how to estimate the expected value of a random variable. Suppose that CF_a is the cash flow brought by financial asset V_a . Due to the uncertainty of V_a , CF_a is also uncertain. To estimate the future cash flow, we use statistical tools to obtain the estimated value CF_a of $\hat{C}F_a$.

According to statistical theory, calculating $\hat{C}F_a$ is a typical estimation problem in statistics: First, CF_a is a random variable, estimating its value is to obtain its future expected value μ_{CF_a} , but in reality, we do not know the probability distribution and parameters of random variables, so we need to use the probability distribution of statistics, namely the sampling distribution, to estimate the parameters of the population distribution. Secondly, the sample value and sample size of sample CF_a are obtained. Based on the relationship between sampling distribution, population distribution and sample size, the parameter of the overall distribution is inferred: the expected value μ_{CF_a} of CF_a . According to the central limit theorem of statistics, a random sample with a capacity of n is selected from a population with a mean of μ and a variance of σ^2 . When n is sufficiently large (usually $n \geq 30$ is required), the sampling distribution of the sample mean approximately follows a normal distribution with a mean of μ and a variance of σ^2/n . This theorem tells us that regardless of whether the original population distribution is normal or not, the sampling distribution of the sample mean will tend to be normal with the increase of the sample size. This solves the problem of estimating expected cash flow.

4. No arbitrage estimation of the value of financial assets

No arbitrage is a very important concept in modern financial asset pricing. If, in A financial market, there are two types of assets or a combination of assets $B(t)$ and $S(t)$, the arbitrage theorem can be expressed as:

Suppose that after the interval T , there are two possible states for each asset, $B(t)$ is $B_1(t+T)$ and $B_2(t+T)$, $S(t)$ is $S_1(t+T)$ and $S_2(t+T)$, and each state occurs with a positive probability. If the normal numbers φ_1 and φ_2 make the asset price satisfy the following relation, then there is no arbitrage opportunity in the financial market.

$$\begin{pmatrix} B(t) \\ S(t) \end{pmatrix} = \begin{bmatrix} B_1(t+T) & B_2(t+T) \\ S_1(t+T) & S_2(t+T) \end{bmatrix} * \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

According to the arbitrage theorem,

$$B(t) = B_1(t+T) \cdot \varphi_1 + B_2(t+T) \cdot \varphi_2$$

$$S(t) = S_1(t+T) \cdot \varphi_1 + S_2(t+T) \cdot \varphi_2$$

This leads to:

$$1 = \frac{B_1(t+T)}{B(t)} \cdot \varphi_1 + \frac{B_2(t+T)}{B(t)} \cdot \varphi_2$$

$$1 = \frac{S_1(t+T)}{S(t)} \cdot \varphi_1 + \frac{S_2(t+T)}{S(t)} \cdot \varphi_2$$

To wit:

$$1 = \frac{B_1(t+T)}{B(t)} \cdot \varphi_1 + \frac{B_2(t+T)}{B(t)} \cdot \varphi_2 = \frac{S_1(t+T)}{S(t)} \cdot \varphi_1 + \frac{S_2(t+T)}{S(t)} \cdot \varphi_2$$

If there is a risk-free asset in the financial market, or a risk-free asset portfolio, that is, $B_1(t+T) = B_2(t+T) = B(t) \cdot (1 + r_{no\_risk})$, then the above formula becomes:

$$1 = \frac{B(t)}{B(t)} \cdot (1 + r_{no\_risk}) \cdot \varphi_1 + \frac{B(t)}{B(t)} \cdot (1 + r_{no\_risk}) \cdot \varphi_2 = \frac{S_1(t+T)}{S(t)} \cdot \varphi_1 + \frac{S_2(t+T)}{S(t)} \cdot \varphi_2$$

Let's say $\overline{P}_1 = (1 + r_{no\_risk}) \cdot \varphi_1$, $\overline{P}_2 = (1 + r_{no\_risk}) \cdot \varphi_2$, then we have $\overline{P}_1 + \overline{P}_2 = 1$. So $\overline{P}_1$ and $\overline{P}_2$ can be thought of as probabilities. If the above formula is true, there must be:

$$\frac{S_1(t+T)}{S(t)} \cdot \frac{\overline{P}_1}{(1 + r_{no\_risk})} + \frac{S_2(t+T)}{S(t)} \cdot \frac{\overline{P}_2}{(1 + r_{no\_risk})} = 1$$

After finishing:

$$\frac{S_1(t+T) \cdot \overline{P}_1 + S_2(t+T) \cdot \overline{P}_2}{S(t)} = (1 + r_{no\_risk})$$

The numerator is similar to the expected price $E^{\overline{P}}[S(t)]$ of asset $S(t)$ at time T , so we have:

$$(1 + r_{no\_risk}) = \frac{E^{\overline{P}}[S(t)]}{S(t)}$$

To sum up, it can be concluded that if there is no arbitrage, the expected rate of return of the asset or asset portfolio in the financial market is equal to the risk-free rate of return, and the risk-free rate of return is obtained, the financial asset or asset portfolio can be priced.

5. Conclusion

There are two dimensions in estimating the value of financial assets. One is the expected cash flow that financial assets can bring, which determines their intrinsic value; the other is that in financial market transactions, financial assets achieve a supply and demand equilibrium without arbitrage, which determines the equilibrium price. Therefore, the value estimation of a financial asset or financial asset portfolio is determined from two dimensions.

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