Forecasting Market Volatility Using High Frequency Data and Mathematical Finance Methods

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Abstract: Market volatility prediction is of great significance in the financial field, and is crucial to investment decision-making and risk management. In order to predict market volatility more accurately, high-frequency data and mathematical financial methods will be combined. High-frequency data provide more detailed market information, while mathematical financial methods provide rigorous models and tools. By combining high-frequency data and mathematical financial methods, it is expected to achieve accurate forecasting of market volatility. Specifically, it is hoped that it can reveal the volatility asymmetry and volatility aggregation of market volatility, and provide real-time and dynamic volatility forecasts. This will help investors and risk managers to better understand and assess market risks, so as to formulate more effective investment and risk management strategies.
Keywords: High-Frequency Data; Digital Financial Methods; Predicting Market Volatility

1. Introduction

Market volatility forecasting is essential for investment and risk management. However, accurately predicting volatility is challenging due to its complexity and variability. High-frequency data, capturing market dynamics over short timeframes, has gained attention from researchers [1]. It provides richer information and enables micro-level analysis of market fluctuations. Yet, analyzing large volumes of high-noise data requires new methods. Mathematical financial methods, such as stochastic processes and partial differential equations, are widely used in financial market modeling [2]. Integrating high-frequency data with mathematical methods enhances volatility forecasts. This combination explores new perspectives and methods for volatility prediction.

2. Models and features of high-frequency data

High-frequency data provides new perspectives and methods for analyzing and predicting market volatility. In Table 1, volatility asymmetry and aggregation are key characteristics. Volatility asymmetry models (e.g., EGARCH, TARCH, GJR-GARCH, APARCH, VS-GARCH) examine the impact of positive and negative information on volatility, including the leverage effect [3]. Estimation methods include maximum likelihood estimation, generalized moment estimation, and Bayesian analysis. Volatility aggregation models (e.g., GARCH-M, EGARCH, STARCH, SWARCH, HGARCH) capture market changes. For instance, the GARCH-M model links conditional standard deviation to expected returns, while the SWARCH model transitions between different ARCH models to capture large market changes and consider aggregation [4].

Table 1 Correlation models and their characteristics for high-frequency data estimation

nature	Model	model features	estimation method
volatility asymmetry	①Asymmetric EGARCH model adopted by Nelson.②TARCH model proposed by Zakoian.③GJR -GARCH model proposed	Model ① analyzes the impact of different news on stock volatility. Model ② adds nominal variables to the conditional variance to distinguish the impact of positive information and	⑴use approximation or simulation The method of constructing the model The likelihood function of and without Conditional

	by Glosten, Jagannathan and Runkle.④APARCH model proposed by Ding, Granger and Engle.⑤VS -G ARCH model proposed by Fornari and Mele.	negative information on volatility. Model ③ adds seasonal items to the GARCH-M model to distinguish positive and negative information. There are different effects of negative shocks on stock price fluctuations. Model ④ has two more parameters than GARCH to study the leverage effect in the stock market. Model ⑤ can describe the asymmetric reverse effect.	moments, e.g. max likelihood estimation (QML), Generalized moment estimation (simulate ML), effective moment estimation method (EMM), etc. ⑵Basic Based on Bayesian principle parameter posterior distribution score Analysis, example: Gibb sampling method in (MCMC) method to estimate the model, etc.
Volatility aggregation	GARCH-M model ①proposed by Engle, Lilien and Robbins. © EGARCH model ② proposed by Nelson.③ STARCH model proposed by Harvey, Ruiz and Sentana.④ Cai, Hamilton and Susmel Proposed SWARCH Model.⑤ HGARCH model proposed by Daccorog et al.	Model ① introduces the conditional standard deviation into the mean equation in order to make the expected rate of return closely related to risk. Model ② avoids the non-negative assumption of parameters. Model ③ requires Kalman filter to be estimated. Model④, the SWARCH model proposed by Cai, Hamilton, and Susmel, assumes several different states of volatility to capture the effect of large market changes. The ARCH model of the GARCH model is converted between them through the Markov chain. Model ⑤ introduces the time scale transformation processing technology in the condition and variance item of the GARCH model.	
long-term memory effects, volatility autocorrelatio n coefficient of double Curve Decay process (5).	① Granger, Joyeus and Hosking proposed up ARFIMA model. ② Baillie, Bollerslev and Mikkelsen came up with FI-GARCH model and Bollerslev and Mikkelsen Proposed F1EGARCH model. ③ Zumbach came up with LM-ARCH model. ④ Beidt et al proposed long-term memory random wave Dynamic (LM-SV) model * ⑤ Ding and Granger 's long-term memory ARCH model. ⑥ Robinson and	Model ① combines fractional difference noise and ARMA model. Model ② explains the heteroscedasticity of sequence changes and long-term memory variability. Model ③ introduces weights into real fluctuations. Model ⑤ ⑥ are primarily designed to reflect the hyperbolic decay of autocorrelation coefficients in the nature of the process.	

	Zaffaroni 's long-term memory Nonlinear moving average model.		

3. Characteristics of Mathematical Finance Methods

Mathematical financial methods, incorporating stochastic processes, probability theory, and partial differential equations, offer a robust theoretical framework and tools for interpreting and forecasting market volatility by rigorously encapsulating the complex dynamics of financial markets.

Stochastic processes are one of the main tools for describing the dynamics of financial markets. For example, geometric Brownian motion is a common stock price model, and the dynamics of stock price S can be expressed as: $dS / S = \mu dt + \sigma dW$, where μ is the expected rate of return, σ is the volatility, and W is the Brownian motion. By solving this stochastic differential equation, we can get the expected change of the stock price in a given time interval [5] .

Probability theory provides tools for quantifying risk and uncertainty in financial markets. For example, value at risk is a common risk measurement method, which $VaR\_ \alpha$ indicates the maximum possible loss of a portfolio under the confidence level of α.

Partial differential equations are widely used in fields such as option pricing. The most famous model in the field of options pricing is probably the Black-Scholes model, which is known for its pricing formula: $C = SON(d1) - Xe^\wedge\{-rt\}N(d2)$, where N is the standard normal distribution function, and d1 and d2 are based on the stock price, strike price, risk-free interest rate, time to maturity, and Volatility is a function of the parameter. Overall, mathematical financial methods, with their theoretical rigor and wide applicability, provide powerful tools for analyzing and predicting market volatility.

4. Combining high-frequency data and mathematical financial methods to predict market volatility

Combining high-frequency data and mathematical financial methods to predict market volatility, the basic idea is to use the model fitted by high-frequency data to update in real time, and then combine mathematical financial methods to predict. Taking the GARCH model as an example, its mathematical representation is as follows:

$$\sigma^2 = \alpha_0 + \alpha_1 \varepsilon^2 (t-1) + \beta \sigma^2 (t-1)(1)$$

Among them, σ^2 represents the market volatility at time t, $\varepsilon^2 (t-1)$ represents the square of the error in period t-1, and $\sigma^2 (t-1)$ represents the volatility in period t-1. When high-frequency data is obtained, a shorter time interval (such as minutes or seconds) can be used to fit the GARCH model to obtain more $\alpha_0, \alpha_1, \beta$ parameter values. Predictions can then be made by updating parameter values in real-time in conjunction with newly acquired data.

Mathematical financial methods like Monte Carlo simulation can optimize forecasts by conducting numerous simulations of future market volatility using recent parameter values. This produces a forecasted volatility distribution, facilitating the calculation of key risk indicators such as value-at-risk and expected loss. This method allows for a more real-time, accurate prediction of market volatility, thereby providing precise risk assessments and decision support for financial market participants. Moreover, the continued refinement of these mathematical financial methods can enhance their adaptability to evolving market environments and emerging high-frequency data types, thereby improving forecast accuracy and practicality.

5. Epilogue

With the increasing complexity and electronicization of financial markets, the importance of using high-frequency data to predict market volatility has become increasingly prominent. Mathematical financial methods provide a rigorous theoretical basis and effective calculation tools to better understand and predict market volatility. For the prediction of market volatility, the combination of high-frequency data and mathematical financial methods provides new possibilities. High-frequency data can reveal more refined market dynamics, while mathematical financial methods can provide estimation of model parameters and calculation of risk measures, thereby realizing real-time and dynamic prediction of market volatility. Despite some progress, there are still many challenges in forecasting market volatility from high-frequency data. For example, the complexity of data cleaning and processing, the real-time update of model parameters, and the computational demands of large-scale simulations.

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