Universe
Scientific Publishing

A Robust Investment Trading Decision Support System Based on Nonlinear TAR Cointegration Theory

Sichen Ye

Financial Engineering Department,College of Economics and Management,China Jiliang University, Hangzhou 310018, China.

Abstract: The current financial market is unpredictable, and traditional investment can no longer meet the systematic hedging investment needs of investors. This investment decision-making system fully considers the complexity of the market and the needs of investors, bringing together multiple investment categories to achieve comprehensive investment, and conducting precise and systematic risk avoidance analysis and robust decision-making. This paper focuses on justifying and exploring the relevant grounded theory of the system.This article takes precious metals as an example for report writing. Innovatively utilizing a precious metal market trading strategy based on threshold cointegration model and risk pricing to construct an arbitrage and hedging investment system suitable for a wide range of investors and enterprises. It also carries out rigorous demonstration and modeling calculations on inter-period and inter-market cross-species arbitrage hedging, which confirms its feasibility.

Keywords: Precious Metal Market; Nonlinear TAR Cointegration Model; Arbitrage And Hedging System

1.Introductory

1.1Macro market background

At present, the global economic growth rate is further slowing down, and global inflation remains at a high level, resulting in an increasing risk of stagflation. The main economies are still tightening their monetary policies, driving the accelerated transition of the world financial cycle. This situation gives rise to an escalation in social financing expenses an a more substantial adverse effec on the economy, thereby amplifying the likelihood of fluctuations in the global financial landscape. The value preservation property of gold is particularly prominent in a turbulent world environment. As an independent fixed asset and special commodity, its own value often increases with the growth of the world economy.

1.2 Original intention of product development

Design a precious metal intelligent investment advisory system platform to fill market gaps and provide practical systems and comprehensive investment channels for investors who lack necessary information and decision support tools. Provide investors with precise services, relevant risk control, and diverse investment advice to meet their hedging and low-risk arbitrage needs in highly uncertain economic environments and high-risk financial markets.

2. Literature review

2.1 Arbitrage

Scholar Qiangsheng (2013) introduced in the article that statistical arbitrage strategies can budget risks and provide risk adjusted returns. Scholars in their research employ statistical arbitrage, a fundamental cointegration model, as Burgess (1999) defined it as an extension of traditional “zero risk” arbitrage through research.Jiao (2013) opines that the current statistical arbitrage techniques mainly utilize cointegration techniques for arbitrage models, due to the fact that the implementation of statistical arbitrage is reliant on the market’s “short selling mechanism”.

2.2 Threshold cointegration

Threshold cointegration increases the difficulty of model identification, testing, and estimation by introducing nonlinear behavior. Balke and Fomby (1997) were the first to propose two threshold autoregressive models, equilibrium TAR and regional TAR, based on their practical economic significance.

Gouveia and Rodrigues (2004) discovered the flexibility of the SETAR model and used it to study threshold cointegration behavior. In China, Tian Zheng and Yang Zheng (2005) conducted relevant research on parameter estimation and threshold testing of threshold cointegration models. Liu Yinxu and Zhang Shiying (2005) validated the superiority of threshold cointegration systems in prediction by conducting threshold cointegration analysis on the Shanghai and Shenzhen A-share indices. Employing the Shanghai and Shenzhen 300 stock index futures' actual trading data, Li Shiwei (2011) sought to refine the existing cointegration theory and construct a model for arbitrage, capable of executing cross period arbitrage. Through the use of threshold cointegration, Wang Long and Chen Shoudong (2012) investigated index arbitrage, ascertained the double threshold value of stock index futures arbitrage, thus establishing the non arbitrage interval.

3. Innovation points

To fill the market gap and adapt to new market demands, the value preservation properties of precious metals are particularly prominent in the environment of maintaining high global inflation levels. For investors who have the need for precious metal arbitrage, hedging operation assistance, and investment hedging, there is a lack of corresponding systems in the market. This product is using a scientifically rigorous nonlinear threshold autoregressive model (TAR) to construct cross variety, cross period, and cross market arbitrage, with a scientifically rigorous logical system; Using innovative investment theory and characteristic risk control system combined with backtesting simulation for in-depth research, constructing a low-risk diversified investment portfolio; Verify its practicality using real products, make reasonable use of asset allocation, investment portfolio, and quantitative trading thinking, and achieve an integrated, efficient, and high win rate precious metal hedging investment system.

4. Partial theoretical foundation

4.1 cointegration theory

Calculating the correlation coefficient measures the strength and direction of the linear relationship between two variables. These coefficients indicate the degree of proximity between two fixed-distance variables and are given in the following formula:

$$r = \frac{\sum_{t=1}^{n}(R_{1t}-\overline{R_1})(R_{2t}-\overline{R_2})}{\sqrt{\sum_{t=1}^{n}(R_{1t}-\overline{R_1})^2\sum_{t=1}^{n}(R_{2t}-\overline{R_2})^2}}$$

Cointegration testing can be used to determine the stability of linear equations between two unstable time series. In this paper, we use the EG two-step method.A linear expression of the long-term cointegration relationship between asset allocation ratios R1 and R2 is that arbitrage is employed to hedge the risk of both assets, with the hedging ratio determined by the equation β Performance, which states that one unit of R1 must be hedged, and a reverse hedging of unit R2.

$$\mathrm{R1} = \alpha + \beta\,\mathrm{R2} + \varepsilon$$

4.2 Threshold Autoregressive Model (TAR)

The threshold autoregressive model describes the motion mechanism of residuals based on cointegration relationships. Should the residual surpass the threshold, a regression equilibrium value will be seen; the residual is within the threshold range and displays random walks.

Arbitrage is only meaningful when the residual deviation from the equilibrium value exceeds a certain critical value, and the arbitrage benefits cover costs and commissions. The mathematical expression for the threshold autoregressive model's upper and lower threshold values is:

$$\mathrm{X_t} = \sum_{i=1}^{k}\{b_{i0} + b_{i1}X_{t-1} + \cdots b_{i,p}X_{t-p} + \sigma_i\varepsilon_t\} I(X_{t-d} \in A_i)$$

This is the critical value in the threshold cointegration arbitrage strategy.

$b_{i\theta}$ is the autoregressive coefficients to be estimated, p being the autoregressive lag order of each system, ε_t is white noise, k is the number of systems in the TAR model, X_{t-d} is the transformation variable, and d is the lag parameter. these are all to be taken into account. In real economic problems, two system and three system TAR models are more common.

The TAR model of the three systems requires two threshold values, expressed as:

$$X_t = \begin{cases} b_{10} + b_{11}X_{t-1} + \cdots + b_{1p}X_{t-p} + \varepsilon_t, X_{t-d} \le \gamma_1 \\ b_{20} + B_{21}X_{t-1} + \cdots b_{2p}X_{t-p} + \varepsilon_t, \gamma_1 < X_{t-d} \le \gamma_2 \\ b_{30} + B_{31}X_{t-1} + \cdots + b_{3p}X_{t-p} + \varepsilon_t, X_{t-d} > \gamma_2 \end{cases}$$

In the above equation γ1, γ2 represents two threshold values.

5. Product design modeling

5.1 Design of cross variety arbitrage of gold and silver

In this part, we use the closing prices of gold and silver futures on the Shanghai Futures Exchange from March 2021 to December 2021 as the objects.

Correlation analysis: Through within sample data testing, it was found that the correlation coefficient between the logarithmic prices of gold and silver futures is 0.977, indicating a strong correlation.

Cointegration test: Perform ADF test on the logarithmic gold and silver futures contract prices, and then make them a stationary time series after passing the first-order difference. Testing of the two further revealed a long-term cointegration bond between gold and silver. Therefore, further cointegration statistical arbitrage can be conducted on the two.

Table 1 EG test results

Value		Prob*
Angle Ranger tau statistical	-11.94433	0.0000
Engle-Granger z-statistic	-133.6545	0.0000

5.2 Establishment of arbitrage model

5.2.1 Threshold cointegration process

Based on the OLS results of gold and silver, a linear regression equation can be obtained:

A unit root test on the residual series reveals that there is no unit root and there is stationarity, which suggests that there is a cointegrating relationship between the gold and silver price series over the sample interval.

According to the regression equation of gold and silver sequences, if one hand of gold futures is purchased, the corresponding silver contract needs to be sold by 9.103 hands; Similarly, to sell one gold contract, one needs to buy 9.103 silver contracts.

Due to the fact that the market does not have an immediate response to divergence in practice, we include a lag variable with a range of values of [0,2] and a threshold variable estimation algorithm complexity of n2. Finally, the upper and lower limits of the non arbitrage interval in the arbitrage strategy were estimated, with threshold values of -2.3903 and 3.2222. There is no arbitrage chance between the two limits.

Draw a residual distribution map of the gold and silver price series based on this threshold value.

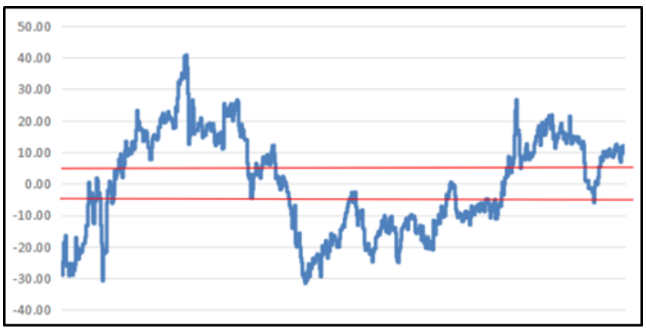

Fig. 1 Residual Distribution of Gold and Silver Price Series

Determination of trading signals:

We estimate that the upper and lower threshold values for the gold and silver sequences are -2.3903 and 3.2222, respectively; The no arbitrage interval is (-2.3903, 3.2222)

1) When $\gamma 1 \leq z\ t \leq \gamma\ 2$, it is a non arbitrage interval and no opening operation will be carried out;

2) When $z\ t < \gamma 1$, when gold is undervalued and silver is overvalued, one should buy one gold futures contract and sell six silver futures contracts. If the residual returns to equilibrium, close the position and complete an arbitrage;

3) When $zt > \gamma 2$,When $zt>\gamma 2$, then the operation in (2) is reversed;

4) In order to effectively control trading risks, stop loss signals need to be added. Assuming that the probability of the residual falling into the stop loss range is 5%, when the trading signal is generated and the position size is established, if $z\ t > z\ 0.975$ or $z\ t < z\ 0.025$, the stop loss will be closed. Z 0.025=-13.87982802, z 0.975=13.80701688.

5.2.2 Design of gold cross period arbitrage

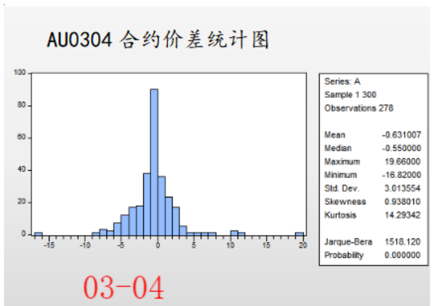

Fig. 2 Arbitrage is most effective with the 03-04 contract

Based on analysis of different month combinations, the distribution characteristics of price differences are analyzed as follows:

1 By comparing the median of the price difference, as well as the median and mean, it can be seen that the distribution of AU0304 is the most symmetrical. The more symmetrical the distribution, the more symmetrical the risk. AU0304 has the lowest risk.

2 The least range of the price difference distribution is AU0304.

3 According to the skewness coefficient, it can be seen that except for AU0304 and AU0607, the skewness coefficients of other price differentials are all negative.The left side of the price difference's dispersion is greater than that of the right, thus increasing the likelihood of arbitrage, and this is evidenced by its skewed distribution.

Based on the above analysis, AU0304 with the lowest arbitrage risk is chosen as an example for cross period arbitrage analysis.

5.3 Cointegration analysis:

5.3.1 Unit Root Test

We select models with constant terms and time trend terms for ADF testing. The p-value of the result surpassing 0.05 implies that the sequence has a unit root and is not static, and thus, in accordance with AIC and SC, the lag term is chosen as 0.

Table 2 Unit Root Test Results for AU03 and AU04

	ADF test statistic	critical value			P-value
		1%	5%	10%	
AU03	-0.779344	-3.453823	-2.871768	-2.572293	0.8229
AU04	-0.624570	-3.453910	-2.871806	-2.572313	0.8616

By observing the time series graph after the first-order difference, we found that the sequence oscillates and fluctuates around the 0 value, without an upward trend.

We opted for a model without fixed terms or temporal trends to carry out unit root tests on DAU03 and DAU04, with a lag term of 0.

5.3.2 Cointegration test

Use two-step cointegration test to analyze AU03 and AU04. Firstly, the EG two-step method was used to test it⍰the sequence R1 symbolizes the residual term of the regression outcome.

Choose the m value that yields the least information criterion value as the lag term, and set the lag period at 1 in accordance with the AIC and SC information criteria. Perform unit root ADF test on sequence R1 with a lag term of 1:

Table 3 Residual R1 ADF test results

	ADF test statistic	critical value			P-value
		1%	5%	10%	
R1	-8.430951	-3.991534	-3.426132	-3.136266	0

It can be seen that sequence R1 does not have a unit root and is a stationary sequence. So AU03 and AU04 have a cointegration relationship, allowing for cross period arbitrage.

5.3.3 Establishment of arbitrage model

After analyzing the price difference, construct an arbitrage model based on the results.The average price difference M of AU0304 is about -0.63; And the standard deviation δ It is 3.01. We set AM as the difference between the price difference and the mean; This can be seen as the degree to which the price difference deviates from the mean.

AM image:

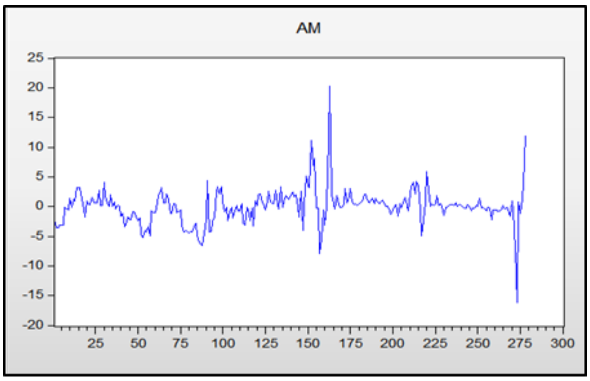

Fig. 3 AM Time Series Diagram

After establishing AM, the first step is to differentiate the price difference, considering the actual issue of handling fees that need to be paid. So when formulating arbitrage strategies, the selection coefficient is excluded Φ Resulting in too little profit, so Φ Take 1 and 1 respectively 25, 1 5. We will (-0 seventy-five δ, 0 seventy-five δ) Within the range of AM, it is considered a normal range, while exceeding this range is considered an unreasonable price difference.

Establish a regression equation to estimate the position ratio of AU03 to AU04 during cross period arbitrage

$$\mathrm{AU04\,t} = -5.855 + 1.015\,\mathrm{AU03\,t} + \mathcal{E}\mathrm{t}$$

1) When AM> Φδ, buy 1 unit of RU03 and sell 1 unit of AU04. When AM returns to (-0.75δ,0 .75δ), or close the position when the contract expires.

2) When AM<-Φδ, opposite direction.

3) If the price difference does not return to the normal range as expected after the position is established, when AM>3δ or AM<-3 δ When the time is up, the stop loss will appear.

6. Conclusion

This product achieves price discovery of precious metal assets in different dimensions through internal design, and uses different financial instruments for rigorous arbitrage and hedging operations.

Overall, for investors or institutions with risk aversion needs, this product can be used for cash management or precious metal equity preservation and appreciation; For aggressive investors and institutions with high return requirements, they can use the correlation between different markets and contracts for high-frequency arbitrage. On the basis of risk control and compliance, they can accept the risk released by the hedger in exchange for high returns.

References

[1] Chen Wei, Zhu Xiaomei Research on Paired Trading of JT1705 and RB1705 Commodity Futures Based on Cointegration Method [J] Operation and Management 2017 (08).

[2] Chen Yiran; A Study on Cross term Arbitrage of Commodities [D] Shandong University 2018.

[3] Fu Jianru, Yuan Qianying, Zhang Wei Selection of Futures Hedging Models: Based on Model (Misconception) Risk and Estimated Risk Perspectives [J] Economic Mathematics 2020 (04).

[4] He He;Research on Arbitrage in the Precious Metals Market Based on Threshold Cointegration Theory [D] Kunming University of Technology 2013.

[5] Lin Ting;A Study on Intertemporal Arbitrage of Shanghai Tianjiao Securities [D] Southwest Jiaotong University 2008.

[6] Qiang sheng; Research on the Application of Statistical Arbitrage in Investment Decision Making of Precious Metal Futures in China [D] East China University of Science and Technology 2013.

[7] Song Changlei; Quantitative Analysis of Arbitrage in Black Industry Chain Varieties [J] Economic Research Guide 2021-11-05.

[8] Xu Zhijia; A Study on the Fluctuation Structure of Price Differences between Gold and Silver [J] Modernization of Shopping Mall 2014-08-10.

[9] Zhou Shanyu; Research on Cross Commodity Arbitrage in Futures Based on Threshold Cointegration Model [D] Southwest University of Finance and Economics 2020.

About author:

Ye Sichen(2002-),male,Han nationality,Shaoxing,Zhejiang,China.