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Theoretical and Empirical Research on Statistical Risk Measurement Methods and Securities Investment Portfolio Models

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Abstract: In order to provide some reference for the research of risk measurement methods statistics and securities investment portfolio models, this paper introduces the theory of risk measurement methods statistics and securities investment portfolio models. The weekly closing price sequence of the Shanghai Composite Index from January 1, 2023 to December 31, 2023 is selected as the sample data, and the application of risk measurement methods statistics and securities investment portfolio models in stock market investment is empirically studied. The conclusion is that the statistical analysis of risk measurement methods and the securities investment portfolio model are highly effective and have a large application space.

Keywords: Risk Measurement Methods Statistics, Securities Investment Portfolio Model, Efficient Market Theory

Introduction:

In 1952, Markowitz first proposed the method of portfolio selection, and modern investment theory officially entered the public eye. After more than 70 years of development, the theory and practice of securities investment portfolio models have been continuously deepened, but there are still many problems, and statistical risk measurement methods are one of them. Therefore, the theoretical and empirical research on risk measurement methods and securities investment portfolio models has significant practical significance.

1. Theoretical Overview

1.1 Behavioral Finance Theory

Behavioral finance theory, proposed by Simeng and Kahneman, emphasizes the irrationality of investors who are influenced by their knowledge and investment skills. The decision support information they collect is restrictive and cannot guarantee complete rationality in the issuance of decisive strategies. For specific individual investors, their decisive strategies are influenced by both their own preferences and those of others, which can easily lead to systematic deviations.

1.2 Efficient Market Theory

The efficient market theory was proposed by Fama and Samuelso, emphasizing that asset prices in an efficient market can reveal all information. Subsequently, Rivers further improved the theory of efficient markets by dividing them into Weak Form, Semi strong Form, and Strong Form.

2. Empirical research

2.1 Method statistics

Value at risk (the maximum loss that a securities investment portfolio can experience in the future at a certain level of confidence) is an important indicator for measuring financial risk, which can be calculated using normal solution methods and Monte Carlo simulation methods. The normal solution method mainly emphasizes that the return and loss functions of the investment portfolio follow a normal distribution. Based on the parameters of the normal distribution, the value at risk can be solved as follows:

$$\mathrm{VaR} = aW(t)\sigma, \quad a = \phi^{-1}(p) \quad \text{Equation -1}$$

In Equation -1, VaR is the value at risk; A is the inverse function; W (t) is the asset value at time t; σ For relative losses; φ- 1 (·) is the inverse function of the standard normal distribution function; P is the confidence level, 95/97.5%/99%. The normal solution method is suitable for solving individual securities. In several securities investment portfolios, weights need to be pre-set, and the distribution parameters of portfolio returns need to be calculated based on historical securities data to derive the value at risk. There are many error factors involved [2].

Monte Carlo simulation method is based on stochastic simulation thinking, using historical volatility data of securities returns to simulate scenarios with high probability of occurrence, providing a basis for estimating the value at risk. The random simulation process is as follows:

$$dS_t = uS_t dt + oS_t dz \text{ Equation -2}$$

In Equation -2, dSt is the random simulation result at time t; U is the instantaneous amplitude of price change at time t; St is the price variable at time t; Dt is a constant; O is the degree of price fluctuation at time t; Dz is a normally distributed random variable with standard deviation.

2.2 Model construction

According to the investment risk statistical method, a mean value at risk securities investment portfolio model is constructed as follows:

$$VaR_t(r) = (1-a)/(1+a)\left[k\left(X\sum X\right)^{1/2} - X^T R\right] + (2a+c)/(1+a) \text{Equation -3}$$

In Equation -3, VaRt (r) is the value at risk of the r securities investment portfolio considering transaction costs and opportunity costs at time t; A is a multiple of the transaction cost equivalent to the investment transaction amount; K is the initial investment price; X represents the proportion of securities investment; T is the investment time interval; C is a multiple of the investment opportunity equivalent to the transaction amount; R is the expected return on the security.

2.3 Sample selection

In the stock market investment, the weekly closing price sequence of the Shanghai Composite Index from January 1, 2023 to December 31, 2023 was selected as the sample data. All erroneous or missing weekly data were removed, and a total of 155 observation samples were obtained to form a new sequence. The minimum value of the Shanghai Composite Index is 0.06654, the maximum value is 0.08235, the mean is 0.00154, the standard deviation is 0.02352, and the variance is 0.0054. Meanwhile, in order to obtain the opportunity cost, non portfolio investment targets such as the S&P 500 index, Dow Jones Industrial Average, Nikkei 225 index, Hang Seng index, etc. were selected, resulting in an opportunity cost of 0.055%.

3. Results Discussion

3.1 Normality test

Assuming that the return on a securities investment portfolio always follows a normal distribution, conduct a normality test. The hypothesis holds that the scatter density of stock index returns is distributed around the slope y=x.

3.2 covariance matrix

The covariance matrix of returns between the Shanghai Composite Index and the S&P 500 Index, Dow Jones Industrial Average, Nikkei 225 Index, and Hang Seng Index is shown in Table 1.

Table 1 covariance matrix of returns for five stock indices

	Shanghai Composite Index	The S&P 500 Index	Dow Jones industrial average	Nikkei 225 Index	Hang Seng Index
Shanghai Composite Index	6.59E-05	4.52E-05	-6.75E-07	6.58E-05	4.25E-05
The S&P 500 Index	2.50E-06	5.78E-04	4.65E-04	3.92E-04	3.90E-04
Dow Jones industrial average	2.93E-06	4.62E-04	5.02E-04	3.55E-04	3.85E-04
Nikkei 225 Index	6.50E-05	4.25E-04	3.55E-04	6.23E-04	4.75E-04
Hang Seng Index	4.04E-05	4.30E-04	3.79E-04	4.78E-04	8.25E-04

In Table 1, “E” is the symbol of Scientific notation, for example, 5.02E-04 is 5.02 times 10 to the power of -4. According to Table 1, among the average returns of the five market indices, the Shanghai Composite Index has a much lower return than other market indices, while the Dow Jones Industrial Average has a higher return than other market indices. When formulating securities investment portfolio strategies, the Dow Jones Industrial Average should be preferred in the market to reduce value at risk and improve returns.

When the transaction cost composed of transaction fees and stamp duty is 0.75%, opportunity cost is 0.055%, and confidence level is 0.01, the optimal strategy for a securities investment portfolio in the market represented by the returns of the Shanghai Composite Index, the S&P 500 Index, the Dow Jones Industrial Average, the Nikkei 225 Index, and the Hang Seng Index can be inferred. According to the difference between opportunity cost and transaction cost, there are also certain differences in the effective frontier equation of the mean value at risk securities investment portfolio model. In order to provide support for investors with different preferences to choose securities investment portfolio strategies under suitable expected returns and confidence levels, the value at risk under opportunity costs and transaction costs should be considered. For example, when the expected return is 0.02, the corresponding value at risk under opportunity cost and transaction cost is 0.9253, and the recommended securities investment portfolio strategy is [22.2352, -31.2514, 11.2351, -0.5254, -6.5824, 3.6514].

Conclusion

In summary, based on the statistical analysis of securities investment risk estimation methods based on value at risk, a mean value at risk securities investment portfolio model that follows a normal distribution is established, and the effective frontier equation and optimal strategy of the model are output. Through further verification of the mean at risk securities investment portfolio model using the Shanghai Composite Index, S&P 500 Index, Dow Jones Industrial Average, Nikkei 225 Index, and Hang Seng Index, it can be concluded that opportunity cost and transaction cost have a direct impact on the effective frontier of the mean at risk securities investment portfolio model. Therefore, opportunity cost and transaction cost should be considered in formulating securities investment portfolio strategies.

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