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An Economic Production Quantity Model under Market Demand Decline and Inflation

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Abstract: Products with the life cycle will eventually enter a decline period. To solve the declining market demand caused by inventory overstocking, and large amount of capital investment in inventory, is a complicated equation. This paper establishes an economic production quantity (EPQ) model in the case of declining market demand and inflation. Assume that (1) the demand rate depends on a linear decline in time, (2) productivity depends on inventory levels, and (3) inflation are fixed. First order linear differential equations are used to construct corresponding mathematical models. The objective is to find the optimal production time to minimize the total average inventory cost within a production scheduling period. A numerical example is given to illustrate the proposed model. Finally, the optimal solution of main parameters sensitivity analysis. These conclusions can help managers to provide effective economic control over production strategies in the face of declining market demand and inflation in the product life cycle.

Key words: EOQ, EPQ, Inflation, Decline.

1.Introduction

Product demand characteristics are different throughout their life cycle. During the introduction period, the demand is relatively small. Company can maintain low inventory levels. During the growth period, demand increases rapidly. If the company maintains low inventory levels, there will be shortages. In the mature stage, if the company maintains incorrect inventory level, it could lead to large inventory and increase inventory cost. Therefore, at different phases of product life cycle, the production time, demand and inventory strategy are also different.

In the decline period, demand of product could be dropped to zero over time. In order to meet the market demand of products in the decline period, this study established EPQ model by assuming that the demand rate depends on the linear decline of time. This model seeks to manage the products during the decline through the strategy of minimizing the total average cost of inventory. As the result, it could help to optimize the enterprise inventory level and production time.

Traditionally, trade between retailers and manufacturers is based on economic order quantity (EOQ) or EPQ models. Many scholars have established many mathematical models to control inventory. Among them, in the production inventory system, the effect of productivity cannot be ignored. To our knowledge, many researchers have attempted to create deterministic EPQ models with variable production rate. To incorporate the production rate, Balkhi and Benkherouf [1] considered an inventory model in which arbitrary production and demand rate depends on the time function. Sridevi et al. [12], Srinivasa et al. [13] developed and analyzed an inventory model with the assumption that the rate of production is random which follows a Weibull distribution. Bhunia and Maiti [2], Perumal and Arivarignan [10] considered two different rates of production in one inventory system. Srinivasa et al. [14], Uma et al. [17] considered the inventory models with stock dependent production rate. Mahata and Goswami [8] considered the fuzzy production rates in their developed EPQ model. In addition, Su and Lin [15][16] developed a production inventory model in which considers the dependence of production rate on the demand and inventory level.

In the process of product life cycle management, enterprise should formulate corresponding inventory strategies according to the demand characteristics in the decline period. Based on characteristics of the inventory strategy, enterprise should formulate the corresponding production strategy. The two strategies affect and interact with each other. This study is based on the above literature. It is practical to assume that production depends on inventory level to establish EPQ model.

In the period of inflation, the inventory holding cost and the opportunity cost of capital occupation will occur when a certain amount of inventory is stored. Therefore, enterprises should fully consider the impact of inflation on inventory and reasonably determine the amount of inventory held. Moreover, countries usually resort to deflation during periods of inflation. Affected by this, the demand for enterprise products in consumption, export, investment and other aspects will be greatly reduced. Therefore, holding an appropriate amount of inventory can reduce the cost of rising prices to enterprises. During periods of high inflation, the adjustment of inventory levels and production is a management measure of inflation. But the increase of inventory will lead to the increase of capital cost. It's necessary to find a balance which to achieve the minimum total average inventory cost.

In the past many authors have developed different inventory models under inflationary conditions with different assumptions. First time it was applied by Buzacott [3] and Misra [7] developed an EOQ model with a constant inflation rate subject to different types of pricing policies. Many other researchers like Hou [5], Hou et al. [6], Nita [9], Chandra et al. [4], Yadav et al. [18], and Yang et al. [19] etc. have considered the effect of inflation in their model. However, most of these studies were limited to the EOQ model to explore the impact of inflation. Sarkar et al. [11] established the EMQ model with price and time dependent demand under the effect of reliability and inflation. The literature on inflation in the EPQ model is rarely mentioned. The existence of inflation in the inventory system may cause the burden of inventory costs. Therefore, EPQ calculated according to the standard model must be expanded to reflect this rate of inflation. Let the enterprise have a suitable mode to develop the production plan, so as to reduce the increase of inventory cost caused by inflation.

In conclusion, the impact of demand, production and inflation rate on inventory model is worth further study. This study attempts to establish the EPQ model under the market demand decline and inflation. Assume that the rate of demand depends on a linear decline in time, that productivity depends on inventory levels, and that inflation is fixed. The first order linear differential equation is used to derive the mathematical formula of the total average inventory cost function. Then Bolzano's Intermediate Value Theorem is used to easily calculate the optimal production time, maximum inventory level and total average inventory cost. In order to illustrate the practicability of the model, an example is given to show the result. Finally, the sensitivity analysis of these parameters to the optimal solution of the established model is discussed. The results provide the manager with the ability to select the appropriate parameter values for the parts that are important.

2.Notations and assumptions

In this section, the EPQ model is defined in the case of market demand decline and inflation. The notations and assumptions used are as follows.

Notations

$I(t)$ inventory level at any time t, $t \geq 0$,

I_m maximum inventory level,

C_s setup cost for each new cycle,

C_i inventory holding cost per unit per month,

r the inflation rate,

T the production scheduling period,

K the total average inventory cost.

Assumptions

1. A single product is considered for only one production scheduling period. Where $T = t_1 + t_2$, t_1 and t_2 are respectively the production time and the use time after production is stopped.

2. Shortages are not allowed.

3. Assume that the demand rate depends on the time of the linear decline. $D(t) = a - b$, where $a, b > 0$. And b is the decline of market demand per unit time.

4. Assume that the production rate depends on inventory levels. $P(t) = \alpha - \beta I(t)$, $\alpha > 0$, and $0 \le \beta < 1$. And β indicates that the ratio of production can be reduced by considering inventory level.

5. Assume that the inventory holding cost at time $t = 0$ is C_i, so it becomes $C_i e^t$ at any time after inflation.

3.Mathematical formulation

The inventory level starts at time $t = 0$ and reaches the maximum inventory level of I_m after the end of time unit t_1. Stop production when the maximum inventory level is reached. At this point, the inventory level continues to decrease as the product is sold, and becomes zero at time $t_1 + t_2 (= T)$. Therefore, the inventory level at any time t can be represented by the following differential equations.

$$\frac{dI(t)}{dt} = P(t) - D(t)$$
$$= (\alpha - a + b\ ) - \beta I(t), \qquad 0 \le t \le t_1 \qquad (1)$$

and

$$\frac{dI(t)}{dt} = -D(t)$$
$$= -(a - b\ ), \qquad t_1 \le t \le t_1 + t_2 (= T) \qquad (2)$$

The first order linear differential equation can be solved by using boundary conditions.

$$I(t) = e^{-\int_0^t \beta dt} \times \int_0^t (\alpha - a + bt) e^{\int_0^t \beta dt} dt$$

$$= \frac{\alpha - a - b}{\beta}\left(1 - e^{-\beta t}\right) + \frac{b}{\beta}, \qquad 0 \le t \le t_1 \qquad (3)$$

and

$$I(t) = -\int_t^T (a - bt) dt$$

$$= a(T - t) - \frac{b}{2}\left(T^2 - t^2\right), \qquad t_1 \le t \le T \qquad (4)$$

When the time reaches t_1, the maximum inventory level I_m can be deduced from (3) and (4).

$$I_m = \frac{\alpha - a - b}{\beta}\left(1 - e^{-\beta t_1}\right) + \frac{b_1}{\beta} = t a_2 - \frac{b}{2}\left(t_2^2 + 2t_1 t_2\right) \qquad (5)$$

Therefore, the relationship between t_1 and t_2 can be related by equation (5).

$$t_2 = \frac{(a - bt_1) - \sqrt{(a - bt_1)^2 - 2b\left[\frac{\alpha - a - b}{\beta}\left(1 - e^{-\beta t_1}\right) + \frac{bt_1}{\beta}\right]}}{b} \qquad (6)$$

In order to consider the market demand decline and inflation, the EPQ model is modified. Modify the total inventory cost function as follows:

$$C_i \int_0^{t_1} I(t) e^t dt + C_i \int_{t_1}^{t_1 + t_2} I(t) e^t dt$$

$$= C_i \int_0^{t_1} \left[\frac{\alpha - a - b}{\beta} \left(1 - e^{-\beta t}\right) + \frac{bt}{\beta} \right] e^{rt} dt + C_i \int_{t_1}^{t_1 + t_2} \left[a(T - t) - \frac{b}{2} \left(T^2 - t^2\right) \right] e^{rt} dt \qquad (7)$$

Hence, the total average cost of the inventory system is

K = setup cost + holding cost

$$= \frac{C_s}{T} + \frac{C_i}{T} \left[\frac{\alpha - a - b}{r\beta} \left(e^{t_1} - 1\right) - \frac{\alpha - a - b}{\beta(r - \beta)} \left(e^{(r-\beta)t_1} - 1\right) + \frac{bt_1}{r\beta} e^{t_1} - \frac{b}{r^2 \beta} \left(e^{t_1} - 1\right) \right]$$

$$+ \frac{C_i a}{T} \left[\frac{e^{r(t_1 + t_2)} - e^{t_1}}{r^2} - \frac{t_2 e^{t_1}}{r} \right]$$

$$+ \frac{C_i b}{T} \left[\frac{t_2^2 + 2t_1 t_2}{2r} e^{t_1} - \frac{t_1 + t_2}{r^2} e^{r(t_1 + t_2)} + \frac{t_1}{r^2} e^{t_1} + \frac{1}{r^3} \left(e^{r(t_1 + t_2)} - e^{t_1}\right) \right]. \qquad (8)$$

The mathematical model above is quite complex. This study use Bolzano's Intermediate Value Theorem to find the optimal value of t_1. First, make $f(t_1) = \frac{dK}{dt_1}$ (see appendix). Suppose $f(t)$ be a function which is continuous on the closed interval $[L, U]$ and such that $f(L)f(U) < 0$. Then there is $t_1 \in [L, U]$ such that $f(t_1) = 0$. Finally, a computer can be used to find the optimal value of t_1.

4.Numerical example

A mathematical example is given to illustrate the practicality of the proposed model. The required parameter values are as follows

$\alpha = 30$ units/month

$C_s = \$100$ for each new cycle

$a = 200$ units/month

$\beta = 0.3$ (the ratio of production can be reduced by considering inventory level)

$b = 10$ units/unit time (the decline of market demand per unit time)

$C_i = \$1$ /unit/month

$r = 0.02$.

The optimal solution of The EPQ model under the market demand decline and inflation can be obtained through the Intermediate Value Theorem.

$t_1 = 1.1451$ months (optimal production time)

$T = 1.8223$ months (production scheduling period)

$I_m = 125.3913$ units (maximum inventory level)

$K = \$120.2635$ (total average inventory cost)

Through the verification of mathematical examples, it is helpful for managers in the face of declining market demand and inflation. The most important parameter value is brought into the model to obtain the optimal production time to reduce the total average inventory cost.

5.Sensitivity analysis

Using the data of numerical example, the variation of 10% and 20% of each side parameter is calculated. Table 1, 2, 3 and 4 summarizes these results.

Table 1. The optimal solution of t_1, T, I_m and K changes with the change of parameter b, leaving all

parameters unchanged.

b	t_1	T	I_m	K
8	1.1712	1.8205	122.0880	118.8304
9	1.1578	1.8210	123.7426	119.5600
10	1.1451	1.8223	125.3913	120.2635
11	1.1330	1.8244	127.0304	120.9416
12	1.1217	1.8276	128.6854	121.5970

Table 2. The optimal solution of t_1, T, I_m and K changes with the change of parameter β, leaving all parameters unchanged.

β	t_1	T	I_m	K
0.24	1.0587	1.7484	128.2537	123.4644
0.27	1.1031	1.7860	126.7148	121.7581
0.30	1.1451	1.8223	125.3913	120.2635
0.33	1.1855	1.8578	124.2239	118.9283
0.36	1.2250	1.8930	123.1796	117.7162

Table 3. The optimal solution of t_1, T, I_m and K changes with the change of parameter r, leaving all parameters unchanged.

r	t_1	T	I_m	K
0.016	1.1508	1.8313	125.9449	120.0386
0.018	1.1479	1.8227	125.6633	120.1509
0.020	1.1451	1.8223	125.3913	120.2635
0.022	1.1422	1.8177	125.1095	120.3752
0.024	1.1394	1.8133	124.8372	120.4872

Table 4. The optimal solution of t_1, T, I_m and K changes with the change of parameter C_i, leaving all parameters unchanged.

C_i	t_1	T	I_m	K
0.8	1.3062	2.0752	140.8004	107.2590
0.9	1.2180	1.9368	132.4243	113.9403
1.0	1.1451	1.8223	125.3913	120.2635
1.1	1.0835	1.7254	119.3693	126.2798
1.2	1.0307	1.6423	114.1487	132.0306

Based on sensitivity analysis, this study can infer the following points.

(1) As can be seen from table 1, the increase of parameter b leads to the increase of optimal production scheduling cycle T, maximum inventory level I_m and total average inventory cost K, while the decrease of optimal production time t_1. In other words, the change of parameter b of market demand decline in unit time leads to the positive change

of T, I_m and K, and the negative change of t_1.

(2) As can be seen from table 2, the increase of parameter β leads to the increase of optimal production time t_1 and optimal production scheduling period T, while the decrease of maximum inventory level I_m and total average inventory cost K. In other words, considering inventory level to reduce the change of productivity parameter β leads to the positive change of t_1 and T, and the negative change of I_m and K.

(3) As can be seen from table 3, the increase of parameter r leads to the increase of total average inventory cost K, while the decrease of optimal production time t_1, optimal production scheduling period T and maximum inventory level I_m. In other words, the change of the inflation rate parameter r leads to the positive change of K and the negative change of t_1, T and I_m.

(4) As can be seen from table 4, the increase of parameter C_i leads to the increase of total average inventory cost K, while the decrease of optimal production time t_1, optimal production scheduling period T and maximum inventory level I_m. In other words, the change of inventory holding cost parameter C_i leads to the positive change of K and the negative change of t_1, T and I_m.

Because the problem involves different parameters, managers face the difficult task of accurately estimating these parameters. The purpose of this result is to provide managers with simple guidelines for determining the production scheduling period. Because managers have a set of parameter values, rather than a specific parameter value. These parameter values can be used to select the best mode of production.

6.Conclusion

In a competitive market, the products in the market demand under the uncertainty of decline and inflation may face high inventory levels. When the product life cycle is in the decline phase, there must be corresponding production strategy to reduce inventory cost. In order to be more in line with the actual situation, this study assumes that the demand rate depends on a linear decline in time, productivity depends on the inventory level, and the inflation rate is fixed. An extended EPQ model is established by using first order linear differential equation. Using the Bolzano's Intermediate Value Theorem and the computer can easily solve the established model. A mathematical example provides a valuable reference for managers to control production time and total average inventory cost. In addition, sensitivity analysis is performed to examine the effect of the parameters. Managers can choose according to the results of these parameters change, the optimal production strategy, so as to achieve the purpose of reducing inventory costs. Future research may consider product demand patterns in different life cycle stages. For example, how to develop corresponding production strategies to maintain the lowest inventory cost during the growth period of the market?

Appendix

Now,

$$f(t_1)=\frac{dK}{dt_1}=-\frac{C_s}{T^2}\times T'$$

$$+\frac{C_i}{T}\left[\frac{\alpha-a-b}{\beta}\left(e^{t_1}-e^{(r-\beta)t_1}\right)+\frac{b}{\beta}t_1e^{t_1}\right]$$

$$-\frac{C_i}{T^2}\left[\frac{\alpha-a-b}{r\beta}\left(e^{t_1}-1\right)-\frac{\alpha-a-b}{\beta(r-\beta)}\left(e^{(r-\beta)t_1}-1\right)+\frac{b_1}{r\beta}e^{t_1}-\frac{b}{r^2\beta}\left(e^{t_1}-1\right)\right]\times T'$$

$$+\frac{C_ia}{T}\left[\frac{e^{r(t_1+t_2)}-e^{t_1}}{r}\left(1+\frac{d_2}{d_1}\right)-t_2e^{t_1}\right]-\frac{C_ia}{T^2}\left[\frac{e^{r(t_1+t_2)}-e^{t_1}}{r^2}-\frac{t_2e^{t_1}}{r}\right]\times T'$$

$$+\frac{C_i b}{T}\left[\frac{t_1+t_2}{r}\left(e^{rt_1}-e^{r(t_1+t_2)}\right)\times T'+\frac{t_2^2}{2}e^{rt_1}+t_1t_2e^{rt_1}\right]$$

$$-\frac{C_i b}{T^2}\left[\frac{t_2^2+2t_1t_2}{2r}e^{rt_1}-\frac{t_1+t_2}{r^2}e^{r(t_1+t_2)}+\frac{t_1}{r^2}e^{rt_1}+\frac{1}{r^3}\left(e^{r(t_1+t_2)}-e^{rt_1}\right)\right]\times T',$$

where $$T'=\frac{(a-bt_1)+(\alpha-a-b)e^{-\beta t_1}+\frac{b}{\beta}}{\sqrt{(a-bt_1)^2-2b\left[\frac{\alpha-a-b}{\beta}\left(1-e^{-\beta t_1}\right)+\frac{bt_1}{\beta}\right]}}.$$

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