The Black-Scholes Model Analysis and Comparison

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Abstract: Mathematical or Quantitative model-based trading is evolving and has become more and more popular nowadays. The Black-Scholes model is one of the most widely used models in option pricing. In this article, the researcher wants to find a better option pricing model compared to the Black-Scholes model.Therefore, the Black-Scholes model can be used in most of the cases in options pricing and the Binary tree model is also a good choice which is more simple than the Black-Scholes model with fewer assumptions. For the BP model, if the user has enough training data for option pricing, it will be a good choice with its excellent learning ability for a short period of predicting. For the volatility smile, the Black-Scholes model's use is one of the reasons for it. The volatility will perform differently with different options. The user can use some hedging skills to deal with it. For example, using different ways to build a portfolio.

***Keywords*：** Black-Scholes；BP Model；

1. Introduction

Options are a financial instrument that is based on an underlying asset like stocks. Options contracts give people the right to buy or sell, which is based on what kinds of contracts deciding by underlying assets. Moreover, the Option price is the only variable in an option contract that changes with market supply and demand. Its level directly affects the profit and loss of buyers and sellers, which is the core issue of options transactions. In the formation and development of the international derivative financial market, the reasonable pricing of options is a major problem that plagues investors. With the application of computers and advanced communication technologies, the use of complex option pricing formulas has become possible. In the past 20 years, investors have transformed this abstract math formula into a large amount of wealth by using the Black-Scholes option-pricing model. And with the model improving, more and more option pricing models have been built and used in the real world like the Binary tree Model. And now, machine learning has been used widely to predict the price of options. Besides, when it comes to option trading, the volatility smile cannot be ignored. Many researchers have spent a lot of time finding the reasons for it. Therefore, the researchers want to compare the Binary tree model and Back-Propagating model which is included in machine learning algorithms with the Black-Scholes model in option pricing, and do more research on volatility smiles.

2. Literature Review

2.1 The development of option pricing

The history of options pricing starts from 1900 when the French mathematician Louis Bachelier deduced an option pricing formula from the assumption that stock prices follow a Brownian motion with zero drift. Since then, many researchers have devoted themselves to this theory. The present paper begins by deleting a few restrictions on the option pricing formula based on assumptions that investors prefer more to less. Because the deleted restrictions are not enough to especially determine an option pricing formula, other assumptions are introduced to test and extend the seminal Black-Scholes model for option pricing (Merton, 1973). When it comes to the Black-Scholes model, it's such a great model which makes the price independent of the utility function. If the Black-Scholes model doesn't appear, then different investors will value the risks differently according to their utility functions. Moreover, it gives the explicit formula to price call and put options. Since the Black-Scholes model appears, the heteroskedasticity of assets returns attracts a lot of attention. The models relating to it appears in the following years including the constant-elasticity-of-variance model by Cox (1975), the jump-diffusion model By Merton (1976), the Binary tree model formalized by Cox, Ross, and Rubinstein (1979), and a bivariate diffusion model for pricing options on assets with stochastic volatilities proposed by Hull and White (1987). There are so many option pricings models that appear and now machine learning is also applied to option pricing.

2.2 The Binary Tree model and the Black-Scholes model

The Black-Scholes equation is a partial differential equation that controls the price of options. When it is impossible to use a clear formula, numerical methods can be used for pricing in the Black-Scholes equation. In the Black-Scholes model, the influence of the expectation was eliminated. McCauley, Gunaratne, and Bassle（2016）find that the generalization of the partial differential equation in the Black-Scholes model to the case of variable diffusion D(x, t) describes a Martingale in the risk-neutral discounted stock price. They proved that the Green function for the types of PDE in the Black-Scholes with diffusion coefficient D (x, t) depending on both x and t describes a Martingale. Besides, the key idea behind this model is to hedge options by buying and selling underlying assets in the correct way, thereby eliminating risks. This type of hedging is called "continuously revised delta hedging" and is the foundation of more complicated hedging strategies such as those engaged in by investment banks and hedge funds.

When it comes to the Binary tree model, it’s a model to estimate the value of an option at time t=0 which provides a payoff at a future date on the basis of the value of non-dividend-paying shares at the future date. This model is building based on there is no arbitrage opportunity in the market, and it assumes that the stock price follows a random walk. Every step for the stock has a certain probability of moving up or down. (Ahmad Dar & Anuradha, 2017)

Comparing these two models, the Binary tree model price the options based on discrete-time, and the Black-Scholes model is based on continuous-time option pricing. Moreover, the Black-Scholes model is built on the basis of many assumptions like the price of an underlying asset following a GBM, no taxes, the constant risk-free rate of interest, and volatility. And the assumptions of the Binary tree model are less than the BS model. This made it closer to the real world’s options.

Therefore, the researcher will compare their ability of option pricing to find which is better converging to the real-world options’ price.

2.3 The reasons and the manifestations of the volatility smile

Rubinstein (1994) related the volatility skew effect to disturbances in the price process of the underlying assets not following a geometric Brownian motion with constant volatility. In the presence of disturbances in the price process, skewness and excess kurtosis in the underlying asset return distributions seem to be the main sources of the volatility smile in option prices. (Vagnani, 2009) However, Vagnani (2009) shows that the Black-Scholes model itself is the basic cause of the volatility smile. According to what he said, the Black-Scholes model performs a sort of regulation of the market itself and traders have to adapt themselves to it. Therefore, some models have been suggested to deal with this problem. For example, the constant elasticity of variance model (CEVM) with non-constant volatility is suggested by Edeki et al. (2016) to modify the Black-Scholes model. In this model, the stock price volatility is a function of the underlying asset price instead of a constant number. This model helps to reduce the known volatility smile effect of the lognormal model.

The researcher wants to use numerical calculation methods to measure the implied volatility and research on the performance of the volatility smile in the real world.

2.4 The application of machine learning in option pricing

Because there are so many assumptions of the Black-Scholes model, a lot of financial engineering models have tried to relax the Black-Scholes model restrictions and improve the empirical results. Some of the most popular approaches are as follows. Firstly, Corrado and Su (1996) or Jarrow and Rudd (1982) build models with statistical series expansion. Secondly, Schroder suggests the local volatility model (1989). And there are many other models like stochastic volatility models suggested by Hull and White, 1987, Heston, 1993 or Schöbel and Zhu (1999), and models with jumps like Merton, 1976, Bates, 1996.（Ivașcu, 2021）However, these models are much more computationally costly, which require a lot of implicit parameters to be calibrated. As a result, to efficiently price financial derivatives fast and accurately, another line of research based on data-driven models has been developed. （Ivașcu, 2021）And machine learning is one of the main tools for data-driven models. Many researchers found that the price of options can be predicted by it. According to Ivașcu (2021), a Neural Network is more precise and computationally more efficient than the Black-Scholes model in option pricing. The neural network model is tested according to the benchmark (ie Black-Scholes) Formula with historical volatility growth rate value. And the results are that the neural network (NN) model gives a better out-of-sample fit for market price observation as well as a lower absolute hedge error than the Black-Scholes model. (Amilon, 2003) And a Support Vector Regression (SVR) model and a Gaussian Process (GP) which are included in machine learning are very performant in terms of option valuation. Park et al. (2014) suggest that GP which is the Bayesian non-parametric model is more accurate in predicting option prices than the NN model and this model also provides the predictive distribution of option prices. Yang and Lee (2011) show that the implied volatility of the next day using one-day KOSPI 200 call options of ELW and then use the Black-Scholes equation as well as predict volatility to deal with option prices. However, there some disadvantages to them. They all need a large quantity of historical data for the training set. (Ivașcu, 2021).

The researcher will choose one of the models in machine learning to observe its ability in option pricing comparing to the Black-Scholes model.

3. The calculation and analysis of the volatility smile

From the answers of the researcher’s tutor, volatility smiles a bit like the result of an unreplaceable risk and these uncertainties get larger for options that are deep in the money or deep out of the money, and this obviously produces the smile. The fact that this is a non-replicable risk and therefore depends on utility is also supported by the fact that the smile depends on the environment.

In order to find the performance of the volatility smile in the chosen period, the researcher uses the newton method and dichotomy method to calculate implied volatility. From table 6 which is from data of the options whose date to maturity is 16 days, the researcher finds that the results of the two methods are very similar. Therefore, the researcher chooses the results calculated by the Newton method to plot the figure4.

In order to show the distribution of the yield calculated by the data from the options which are used to calculate implied volatility, the researcher calculates the mean, standard deviation, and skewness, kurtosis to describe the data features. From table 4, the kurtosis for each kind of option is larger than 1 a lot, which means they all have the steep peak and the skewness is all positive which means that their distribution is all right-biased. Then figure5’s blue color distribution is the random normal data sample which has the same mean and variance as the 16-days kind options data plot, we can find that the red one plot by options’ data is on the right-hand side of the normal distribution and slimmer, which is coincides with the conclusions above.

4. Conclusion & Recommendation

In this article, the researcher compares different models to the Black-Scholes model and finds it still performs very well comparing to the Binary tree model and BP model. When it comes to the Binary tree model, it is created with discrete-time. In the real world, options trading is in discrete time. However, the Black-Scholes model is with continuous time. Normally, the discrete-time assumption for models is closer to the real world, nevertheless, the Black-Scholes model performs very well with continuous-time assumption. Moreover, although the Black-Scholes model is built with many assumptions and these bounds make the market become ‘incomplete’ as some of the risks remain and can’t be replicated, it still performs excellently in option pricing. For the BP model, with limited data, it can't perform better than the Black-Scholes model in predicting options' price. Therefore, if you want to predict options' price, the Black-Scholes model and the Binary tree model are all good choices if you don't have enough data and if you have thousands of or more data, you can try the BP model to predict. Besides, the researcher understands the volatility smile deeper by observing its yield distribution and features of data. We can observe the difference between the yield distribution with the normal distribution and find the fat tail. The researcher will try to find some ways to hedge the tail risk and volatility smile in the following time.

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