Consider the Mean-Variance Optimal Risky Portfolio with Maximum Sharpe Ratio

Jiangquan Wu, Senrong Chen, Zhiyong Zhong
Nanfang College Guangzhou, Guangzhou 510970, China.

Abstract: In order to make the classical mean-variance model more applicable, the maximum Sharpe ratio is introduced to optimize the mean-variance model, and the average weighting method is compared with the fundamental strategy weighting method. The results show that the mean-variance model considering the maximum Sharpe ratio has good operability and practical value.
Keywords: Mean-Variance; Model Sharpe Ratio; Optimization

1. Model overview and related concepts

Markowitz's portfolio theory is based on maximising the expected utility of terminal wealth ina single time period and terminal wealth. A single period is when an investor holds an asset for a defined period, holding a security at the beginning of the period and selling it at the end. This simplifies the discounting of a series of cash flows and the calculation of compound interest.

The assumption of maximization of expected utility of terminal wealth is different from that of expected terminal wealth. Because the maximization of wealth itself is not the goal of investors, and the concept of utility includes both the expected value of wealth and the uncertainty of obtaining such expected wealth, that is, the maximization of venture utility is the real goal pursued by investors

1.1 Assumptions of the Markowitz model

The stock market is efficient. In other words, this market is a market with fully open information, fully transmitted information, fully interpreted information and no information delay.

Investors are rational individuals, subject to the behavior of dissatisfaction and risk aversion; The variables that influence investment decision are expected return and risk. At the same risk level, investors prefer the asset portfolio with higher return. At the same return level, they prefer the asset portfolio with lower risk.

Investors evaluate assets and asset portfolios in terms of mean and variance criteria within a single period. This premise assumes that the yield of implied securities is normally distributed, that is, the yield of securities is a random variable with a certain probability distribution, and generally it follows a normal distribution. The characteristic of normal distribution is that the change law of random variables can be completely determined by two parameters, namely, expected value and variance. Under the assumption that the rate of return obeys normal distribution, the expected rate of return and risk of investors investing in this security can be described by expected value and variance.

Assets have infinite separability. On the basis of the above assumptions, by revealing the feasible set of asset portfolio and separating the effective set of asset portfolio, combined with the utility indifference curve of investors studied in the previous chapter, the optimal choice of investors is finally obtained, which is the logical context and core content of Markowitz's portfolio theory.

1.2 Feasible set of risky assets

By giving the feasible set of risky assets and separating the efficient set from it, it is another basic tool to determine the investor portfolio theoretically.

A feasible set of a risky asset is the set of expected returns and variances of all possible portfolios in a capital market that may be affected by a risky asset. The relationship between expected return and standard deviation of all possible portfolios is plotted on the expected return - standard deviation coordinate plane, and the feasible set is represented on the closed curve and its inner region.

1.3 Indifference curve

For a particular investor, given a portfolio of securities, according to his preference attitude towards expected rate of return and risk, and according to the requirement of risk compensation for expected rate of return, a series of portfolios with the same degree of satisfaction (no difference) can be obtained. All of these combinations form a curve in the mean-variance (or standard deviation) coordinate system, which is called an indifference curve for the investor.

The satisfaction degree of combinations on the same indifference curve is the same; The higher the position of the indifference curve, the higher the satisfaction degree of the combination on the curve. The indifference curve satisfies the following characteristics:

(1) The indifference curve slopes to the upper right;

(2) The indifference curve gets steeper as the risk level increases;

(3) Indifference curves do not intersect each other;

(4) The indifference curve represents an individual investor's personal assessment of the equilibrium between expected return and risk, that is, the indifference curve is determined subjectively and the shape of the curve varies from investor to investor.

1.4 Optimal choice of investors

For various alternative risky assets or securities, the efficient frontier can be determined if the expected rate of return and the variance-covariance matrix are known. Investors choose a certain point on the efficient frontier to make investment decisions according to different personal preferences. Since the efficient frontier is convex on the top and the utility curve is convex on the bottom, the two curves must be tangent at a certain point. The tangent point represents the optimal combination that should be selected in order to achieve the maximum utility.

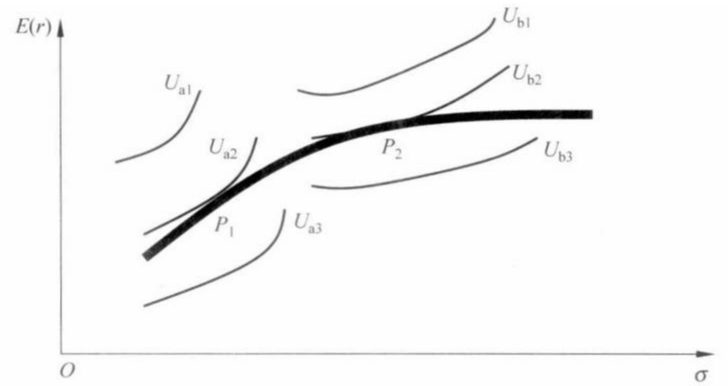

Figure 1. Determination of optimal investment portfolio for investors

Different investors will choose different areas on the efficient frontier of the asset portfolio. Investors with higher risk aversion will choose portfolios close to the end points; The less risk-averse investor will choose the portfolio at the top right of the endpoint.

2. Study on mean-variance model when Sharpe ratio is maximum

The hypothesis of Markowitz mean-variance model has some limitations in the application of Markowitz's investment portfolio: (1) The limitation of risk measurement methods. The classical risk measure factor of mean variance model is variance. In our country, securities investment risk should include the probability of loss, the amount of possible loss, the uncertainty or variability of loss and the high volatility of return. (2) Limitations of theoretical assumptions. The assumption of the mean - variance theory is very strict, which makes it out of line with the reality, thus affecting its effectiveness. (3) The limitation of the theoretical premise. This theory only depend on the mean and variance of two parameters to describe risk asset characteristics, and calculations are based on historical data, and the securities market in China is best known for its severe volatility and volatility, the accuracy of the evaluation work of securities put forward a huge challenge, also make the fund investment managers to assume the risk of a lapse in judgment. (4) Unlimited capital scale is difficult to achieve in reality. Mean-variance theory has no quantity and capital limitation on asset portfolio, but in actual investment, investors' capital is limited. (5) Poor timeliness and flexibility. When calculating the parameters of the mean-variance theory, the sample length needs to be large enough to ensure the accuracy of the parameter estimation. However, the sample data can only reflect the risk situation in the corresponding period. When the sample length is too large, it will often lead to the poor timeliness of the information reflected and the insufficient estimation of the recent changes.

Therefore, in order to further enhance the applicability of the mean-variance model in the field of investment, this paper introduces the Sharpe ratio to optimize the Markowitz mean-variance model based on the existing research results of model optimization in the field of investment science.

Sharpe ratio maximum optimization model is introduced. Take the average rise and fall of stocks in the past ten years as the expected return rate, change the weight of investment assets under the fixed risk level to get the maximum Sharpe ratio, so as to maximize the portfolio return rate.

The optimal model of risky portfolio considering the maximum Sharpe ratio is:

$$\max \ \text{SharpeRatio} = \frac{E(R_P) - R_f}{\sigma_P}$$

$$\min_{(w_1, w_2, \cdots, w_n)} \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

$$\sum_{i=1}^{n} w_i \bar{r}_i = \bar{r}_p$$

$$\sum_{i=1}^{n} w_i = 1$$

$$w_i \geqslant 0, \ i = 1, 2, \cdots, n$$

3. Empirical research

This article selects ten stocks listed on the Shanghai stock exchange (huaxia happiness, guizhou maotai, shandong gold, conch cement, yili corporation, Shanghai pudong development bank, Shanghai airport, citic securities, poly real estate, China unicom) in October 2022 and October 2012 - day trip when calculating the average return rate of data from CSMAR taian database, as shown in Tzble 2.

Through data processing, the covariance matrix between the returns of each stock is obtained, as shown in Table 2.

The most important step below is to solve the maximum Sharpe ratio. In this paper, the yield of Chinese bond and national bond is selected as the risk-free interest rate. Since December 31, 2001, the total index of Chinese bond and national bond (wealth sub-index) has increased by 109.92%, and the annual yield is about 1.25%, so the risk-free interest rate is 0.005%. By solving the optimal model, the weight of each stock is obtained, as shown in Table 3.

Other asset weight selection methods. (1) Average weight method. A simpler weighting method, investment, distributes investment funds equally among promising assets. (2) Fundamental strategy weighting. To calculate the fundamental value of each stock, first, calculate the average of the company's operating income for the past five years, the average of the company's cash flow for the past five years, the average of the company's net assets for the past five years, and the average of the company's total dividends for the past five years using the annual report data for the past five years. Then, Computing operating income of the sample space of all stock as a percentage of total revenues, cash flow of the sample space all shares the percentage of the sum of cash flow, net assets of the sample space all stocks accounted for the percentage of the total net assets and dividends all stock dividend as a percentage of total sample space, the final weight obtained from the above 4% arithmetic average.

The weights of the ten stocks selected in this paper are calculated according to the above two asset weight selection methods, and the return rate of their portfolio from September 2021 to September 2022 is calculated, as shown in Table 4. Compared with the return rate, the asset weighting method proposed in this paper is higher than the average weighting method and fundamental strategy weighting. The average weighting method is simple to calculate and fully diversifies the risk, but it does not consider the return and risk of each stock. The weighting of fundamental strategy is in line with investment logic and the weighting is related to the fundamentals of the company, but the factor has short-term failure risk. The average cash flow of Citic Securities in the past five years is the highest among the ten stocks selected in this paper. From February 2022 to mid-May 2022, the stock price decreased by about 28% due to rights offering and A-share market shocks. And according to the method of fundamental strategy weighting, Citic Securities still chooses a higher weight.

Table1. Average rate of return(%)

	600340	600519	600547	600585	600887	600000	600009	600030	600048	600050
Average rate of return	0.02828	0.12876	0.02420	0.08034	0.11243	0.05090	0.09646	0.07800	0.08169	0.02753

Table2. Covariance matrix

	600340	600519	600547	600585	600887	600000	600009	600030	600048	600050
600340	0.07693	0.00024	0.00017	0.00431	0.00141	0.00216	-0.00059	0.00381	0.00303	-0.00039
600519	0.00024	0.04112	-0.00047	-0.00052	0.00084	0.00229	-0.00128	0.00232	-0.00036	-0.00151
600547	0.00017	-0.00047	0.06739	-0.00087	-0.00196	0.00033	-0.00250	0.00404	-0.00169	-0.00115
600585	0.00431	-0.00052	-0.00087	0.04934	0.00106	0.00740	0.00415	-0.00012	0.01019	0.00465
600887	0.00141	0.00084	-0.00196	0.00106	0.05183	0.00167	0.00084	-0.00152	0.00249	0.00131
600000	0.00216	0.00229	0.00033	0.00740	0.00167	0.02920	0.00325	0.00059	0.00413	0.00299
600009	-0.00059	-0.00128	-0.00250	0.00415	0.00084	0.00325	0.05013	0.00079	0.01520	0.01229
600030	0.00381	0.00232	0.00404	-0.00012	-0.00152	0.00059	0.00079	0.06335	-0.00011	-0.00066
600048	0.00303	-0.00036	-0.00169	0.01019	0.00249	0.00413	0.01520	-0.00011	0.06534	0.01796
600050	-0.00039	-0.00151	-0.00115	0.00465	0.00131	0.00299	0.01229	-0.00066	0.01796	0.05069

Table3. Stock weights obtained from the model in this paper

	600340	600519	600547	600585	600887	600000	600009	600030	600048	600050
Weight	0.04322	0.21647	0.07399	0.09944	0.15380	0.11510	0.12285	0.09583	0.04248	0.03681

Table4. Portfolio rate of return for different methods

	600340	600519	600547	600585	600887	600000	600009	600030	600048	600050	Portfolio rate of return
Stock returns over the past year	-0.07553	0.18772	0.03121	-0.06342	0.09709	-0.14735	0.35642	-0.37900	0.51443	-0.03877	
Model weight in this paper	0.04322	0.21647	0.07399	0.09944	0.15380	0.11510	0.12285	0.09583	0.04248	0.03681	0.05924
Average position mode weight	0.10000	0.10000	0.10000	0.10000	0.10000	0.10000	0.10000	0.10000	0.10000	0.10000	0.04828
Fundamental strategy weighting weight	0.00000	0.22800	0.12213	0.16580	0.16264	0.00000	0.11497	0.38904	0.21617	0.20110	0.04883

4. Conclusion

This paper first introduces the relevant content of Markowitz mean-variance model, and takes this as a theoretical basis. By using the actual data of 10 stocks listed in Shanghai Stock Exchange from September 2012 to September 2022, Data processing in EXCEL is implemented to obtain the weight of each asset when Sharpe ratio is maximum, and an optimal risk portfolio is obtained. Compared with average weighting method and fundamental strategy weighting method, it is found that the weight selection method proposed in this paper has a higher rate of return.

For investors, the method in this paper has good operability and practical value. The optimal risk portfolio can be obtained through the data of stock's past rise and fall and the model in this paper.

References

[1] Li XF. Portfolio Management [M]. Beijing: Tsinghua University Press, 2015:20-26.

[2] Sarenger G. Markowitz mean-variance model and its application [J]. Journal of Jining Teachers College,2007(4): 30-31.

[3] Wei HY, Deng W. Limitations of Markowitz mean-variance Theory and its Applicability in China [J]. Southern Finance.2004(10): 31 ~ 32.

[4] H. Markowitz. John Wiley&Sons Inc Portfolio selection:Efficient Diversification of Investments. 1991.