

# How, Why, and When the Current Real Market Crude-oil Price Differs from the Theoretical Price by the Black-Scholes Model

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Abstract: Real-life crude oil pricing does not strictly follow the traditional and authoritative Black-Scholes pricing model in many cases. In fact, the Black-Scholes model has its own limitations and is challenged every now and then. How the prices derived from the Black-Scholes model really differ from or are similar to the actual market prices and what the causes of the difference between theory and reality are (factors that are remarkably affecting real-world oil pricing) will be explored in this research. This research analyzes the global oil pricing market and studies concrete cases from the CME Group, India NSE stocks, etc. The case study shows the situations when the Black-Scholes model is applicable and not applicable in real market pricing, which leads to the discussion of what actually deviates oil prices from the Black-Scholes model at the global level. This discussion is greatly dependent on the summary of oil prices and relevant events during the past decades.

Keywords: Black-Scholes Model; Crude-Oil; Pricing

# Introduction

This research investigates how crude oil is priced in nowadays real-market, how the authoritative pricing model – Black-Scholes model is challenged in nowadays context, what differences lie between the real-world oil prices and theoretical prices calculated from the Black-Scholes model (especially shown by cases-study), and why this gap between reality and theory exist.

The well-known Black-Scholes model is challenged every now and then, and the limitations of Black-Scholes model have been investigated in plenty of research. However, there is not much research regarding the application of Black-Scholes model in the crude-oil pricing field. Therefore, figuring out the applicability of Black-Scholes model in oil pricing can give further insights into the oil-pricing field.

Cases, when the Black-Scholes model is applicable or not, will be studied and the difference between theoretical Black-Scholes price value and real market price value will be identified and quantified by some calculation results. Further, to explore why this difference exists, historical prices with essential events happening at critical points (for example, turning points) for the past 50 years will be studied. Hopefully, the historical events that were influencing oil pricing provide us with insights about what is really affecting crude oil prices worldwide.

### Literature Review

The Black-Scholes model is popular, recognized, and widely used in nowadays pricing field. It is commonly used by option traders to determine how much a call or an option (European call option, for example) should be priced fairly in theory based on Brownian motion, several non-real-life assumptions, and six variables - volatility, option type, price of underlying, time, strike price, and the risk-free rate.<sup>[3]</sup> However, the considerable limitations of the Black-Scholes model exist due to its non-realistic assumptions of the pricing market (shown below)-(D Teneng, 2011)

- O Volatility is a constant.
- We cannot precisely and even cannot be able to predict the direction of the market.
- o Underlying stock is normally distributed.
- o constant interest rates
- o People do not pay dividends from the underlying stock during the option.
- O No commission costs and no transaction costs exist.
- o European-style options are only conducted on the expiration day.
- o It is able to sell and buy any given stock or options at any amount and at all times. [4][5]

Crude oil trading is recognized widely as an essential aspect impacting the global economy. The oil futures and options contracts are one of the deepest and most influential futures and options markets worldwide.

(Miao, H. et el., 2017) Real market prices and concurrent economic, financial, and geopolitical events are closely related. Some of the most influential parameters are lagged-WTI price, steel production, ISM and Kilian index (which are called demand factors), CRB rind index (which are commodity market factors), the dollar index (the financial factor), and terrorist attacks in certain areas such as the Middle East and northern Africa (the geopolitical risk factor). It is found that the decline since mid-2014 in oil prices can result from the combination and interaction of demand, commodity market, and other financial variables. Covid-19 (political and economic policy towards covid) also has impacts on the oil market (be further convinced). The COVID-19 pandemic has weakened oil demand severely so it only permits slow recovery. It is concluded that the demand weaknesses can curb oil price rises for almost or even more than three or four years in some researches. Seeking and identifying important factors that drive the changes in oil prices can be helpful to a better understanding of Black Sholes model.

Limitations of Black-Scholes model are from time to time mentioned by researchers and modified pricing models are proposed in many papers but there is not much research looking into the application of Black-Scholes model in the crude-oil field since oil prices are determined and impacted by too many factors in the real market. However, identifying how and when those factors affect the accuracy of the B-S model in the oil field still helps integrate the studying of Black-Scholes model and nowadays oil pricing.

# Research methods

A quick overview of real-market crude-oil prices (throughout the past 70<sup>[2]</sup> and 10<sup>[1]</sup> years as examples) is shown below —

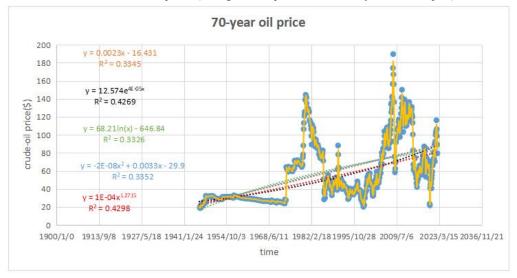


Figure 1: real-market crude-oil prices in the past 70 years



Figure 2: real-market crude-oil prices in the past 10 years

In the past 70 years, the overall prices went through a bunch of stages and fluctuations (stable before 1970, sharply increased in the following 10 years, then dropped, volatile but relatively stable, increased to a record-breaking peak value, then dropped sharply and then volatile) but went up in general. In the latest 10 years, the prices went down and then up but decreases a bit in general.

The ups and downs in the graphs obviously differ from the theoretical values calculated from the Black-Scholes model (especially at the sharp corners in the graph because the calculated B-S price graph is a smooth curve) and the existence of these differences is a good reason and start to investigate the inapplicability of the Black-Scholes model.

A case study when the B-S model is still appliable

Although the theoretical prices calculated from the B-S model are different from the real oil prices from time to time, the B-S model is still authoritative and applicable for some nowadays cases and organizations.

A case study published by Floris Uleman in 2016 compares the option prices calculated from Black-Scholes with the real option prices announced by CME from November 30, 2015 until June 13, 2016.

The calculated results are shown below

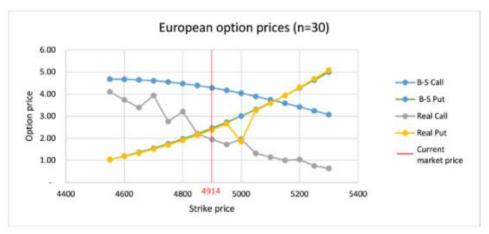


Figure 3: 'call' and 'put' options prices of B-S model and reality for a sample of 30 observations[7]

The "B-S Call" and "B-S Put" curves are the European option prices derived from Black-Scholes theory; the "Real Call" and "Real Put" curves represent the option prices recorded for CME group in the real market. "n = 30" indicates that all calculations and analyses are done for a sample of 30 observations.

Examples when the B-S model is challenged or not appliable

India NSE stocks

10 popular Indian NSE stocks are tracked, calculated, and analyzed as samples to explore the applicability of Black-Scholes model in the context of Indian stock market using the method BSOPM. (Calculation and results from Srivastava, A., & Shastri, M.

(2020)).

Part of the calculated results and market prices of the stocks are shown below.[8]

Table 1: for call options

No.	Date	Cement Industry								Govt Po	wer Sector			Banking Sector (Private)						
		ACC		Ultra Cement Company		Ambuja Cement		ONGC		NTPC		BHEL		AXIS Bank		HDFC Bank		Federal Bank		
		Fair	Actual	Fair	Actual	Fair	Actual	Fair	Actual	Fair	Actual	Fair	Actual	Fair	Actual	Fair	Actual	Fair	Actual	
1	5-Mar	139.3	259	271.67	536	22.5	38.25	14.6	38	5.7	5.15	12.05	25.05	56.7	145.4	50.3	116.2	6.96	15.7	
2	6-Mar	130	259	302.99	536	18.4	38.25	14.4	38	5.65	3.9	10.15	25.05	50.8	145.4	3978	116.2	6.182	15.7	
3	7-Mar	113.9	259	284.49	536	14.61	38.25	11.4	38	5.66	3.9	7.938	25.05	46.9	145.4	34.1	116.2	5.051	15.7	
4	8-Mar	91.54	259	279.18	536	15.16	38.25	10.4	8.3	6.57	3.9	8.387	25.05	52.8	145.4	40.88	116.2	6.955	15.7	
5	9-Mar	82.12	259	258.75	536	14.48	38.25	10.2	8.3	5.99	3.9	8.381	25.05	41	145.4	39.43	116.2	7.534	15.7	
6	12-Mar	99.08	259	321.12	536	17.78	38.25	12.8	11	10.1	3.9	8.878	25.05	50.7	145.4	43.68	116.2	7.674	15.7	
7	13-Mar	109.8	259	309.67	536	16.21	38.25	12.5	11.25	8.77	3.9	8.794	25.05	60.6	145.4	40.01	116.2	8.999	15.7	
8	14-Mar	138.2	259	346.33	536	19.77	38.25	10.2	10	8.9	3.9	8.43	25.05	64.4	145.4	40.94	116.2	7.877	15.7	
9	15-Mar	130.3	259	311.93	536	18.45	38.25	9.92	8.5	8.96	3.9	8.826	25.05	58.5	145.4	47.14	116.2	6.948	15.7	
10	16-Mar	102	259	210.33	536	13.14	38.25	7.77	7.65	5.71	3.9	8.26	25.05	52.4	145.4	34.84	116.2	6.653	15.7	
11	19-Mar	96.01	259	163.94	536	10.55	38.25	7.32	7.65	6.72	3.9	6.118	25.05	49	145.4	30.42	116.2	4.847	15.7	
12	20-Mar	89.35	66	142.9	536	9.987	38.25	5.67	5.85	6.28	3.9	6.317	25.05	50.1	145.4	27.04	13.8	5.106	5.4	
13	21-Mar	91.16	66	175.04	536	9.539	38.25	6.37	5.6	8.8	3.9	5.526	6	54.2	145.4	33.03	17.1	4.864	6	
14	22-Mar	83.91	66	132.72	536	8.543	38.25	8.34	8.5	9.16	3.9	4,4	5.1	47.2	145.4	35.6	17.75	4.741	5	
15	23-Mar	69.02	66	108 04	536	7 867	67	7 28	71	8 42	39	4 345	45	32.7	33	25 38	12.5	4 559	48	

Table 2: For put options

No.	Date	Cement industry						Govt Power Sector							Banking Sector (Private)						
		ACC		Ultra Cement Company		Ambuja Cement		ONGC		NTPC		BHEL		AXIS Bank		HDFC Bank		Federal Bank			
		Fair	Actual	Fair	Actual	Fair	Actual	Fair	Actual	Fair	Actual	Fair	Actual	Fair	Actual	Fair	Actual	Fair	Actua		
1	5-Mar	17.43	6.9	89.95	59.5	4.03	1.4	2.23	1.55	5.67	1.75	0.808	2.9	7.14	1.2	80.22	28.85	3.216	3.05		
2	6-Mar	19.28	6.9	75.061	49	5.382	1.4	2.23	1.55	5.61	1.75	1.124	2.9	8.48	1.2	93.78	28.85	3.606	3.05		
3	7-Mar	23.55	6.9	80.398	49	7.092	1.4	3.17	4.5	5.5	1.75	1.681	1.55	9.41	1.2	102.1	28.85	4.344	3.05		
4	8-Mar	31.76	6.9	80.589	49	6.635	1.4	3.53	4.5	4.64	1.75	1.496	2.7	7.48	1.2	89.05	28.85	3.015	3.05		
5	9-Mar	35.74	6.9	87.448	49	6.855	1.4	3.56	4.5	5	1.75	1.457	1.2	11	1.2	89.79	28.85	2.662	3.05		
6	12-Mar	25.56	6.9	57.995	49	4.845	1.4	2.32	4.5	2.43	1.75	1.202	1.2	7.12	1.2	78.99	28.85	2.407	3.05		
7	13-Mar	21.09	6.9	59.79	49	5.372	1,4	2.33	2.25	2.9	1.75	1.183	1.2	4.82	9.15	82.7	28.85	1.85	3.05		
8	14-Mar	13.22	6.9	47.542	49	3.878	1.4	3.1	2.25	2.76	1.75	1.236	1.2	4.04	9.15	79.77	28.85	2.196	3.05		
9	15-Mar	14.4	6.9	55.437	49	4.2	1.4	3.14	2.25	2.66	1.75	1.097	1.2	4.81	9.15	70.06	28.85	2.535	3.05		
10	16-Mar	21.65	6.9	95.487	49	6.483	1.4	4.18	3.45	4.49	1.75	1.198	1	5.82	6.65	85.95	28.85	2.608	2.9		
11	19-Mar	21.6	6.9	117.03	49	7.637	1.4	4.13	3.9	3.46	1.75	1.754	2	5.9	7.8	88.43	28.85	3.456	3.25		
12	20-Mar	23.34	6.9	130.88	106	7.868	6.9	5.26	5.1	3.64	1.75	1.62	1.95	5.43	9	93.19	28.85	3.183	3.8		
13	21-Mar	21.85	6.9	103.02	106	8.016	5.7	4.55	5.1	2.25	1.75	1.894	2.15	4.41	6.5	80.16	28.85	3.26	3.5		
14	22-Mar	23.91	6.9	133.09	115	8.667	5.7	3.2	3.3	2.04	1.75	2.435	2.6	5.61	7.1	74.28	28.85	3.254	3.45		
15	23-Mar	30.16	69	155 15	115	9 087	5.7	3.68	41	224	19	2 395	265	9 92	11.6	90.65	28.85	3 291	3.8		

The comparison between "fair" and "actual" prices indicates that there does not seem to have a significant association between the actual real-market prices and the stock prices calculated theoretically from the Black-Scholes model. Therefore, in this case, the Black-Scholes model has low applicability and does not act as revealing and relevant to real-market pricing.

Other recent challenges - Negative oil price

U.S. oil prices dropped below zero record-breakingly on April 20 - the contract of May 2020 futures for West Texas Intermediate that the price went from \$18 per barrel to about -\$37 per barrel.<sup>[9]</sup>

In the case of negative prices, Black-Scholes model is greatly challenged since this traditional model operates on the assumption that oil prices can't go below zero. People have to switch to an alternative model such as the Bachelier model for options pricing (as CME Group Inc did) and rewrite risk models.<sup>[10]</sup>

Factors that affect the accuracy of the B-S model and real oil market price in the past decades (1970 – 2022)

# Crude oil prices and key geopolitical and economic events

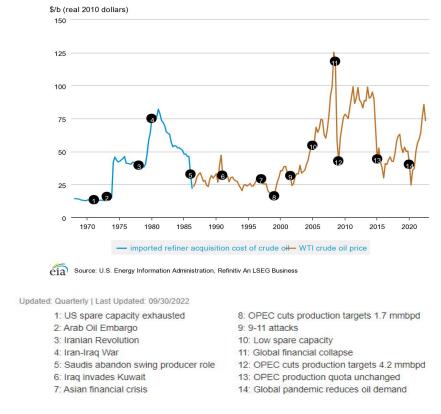


Figure 4: crude oil prices and key events  $(1970 - 2022)^{[12]}$ 

A more detailed table relating crude oil prices to key events (1970 – 2022) can be found at https://www. thebalancemoney.com/oil -price-history-3306200.<sup>[11]</sup>

Oil prices kept relatively steady in the 20<sup>th</sup> century until the 1970s. Then the oil prices were significantly driven up and down by political, economic, environmental, military events, and other factors. All kinds of factors (events) are listed above.

### Results

Calculated B-S model prices and real market prices for the CME group from November 30, 2015 until June 13, 2016, can be said to be consistent. Both 'Call' curves are negatively sloped, whereas both 'Put' curves are positively sloped so the trends are consistent for reality and theory. The curves of "B-S Put" and "Real Put" are very much identical which suggests that the Black-Scholes model is likely to be used as the underlying model by the CME group for option pricing. However, the "call" curves bear less consistency which could originate from external factors (political, economic, environmental, etc.)

However it is not the case for the Indian stock market - the theoretical (B-S model) and actual prices of the stocks are not much related. The Black-Scholes model has low applicability in this case.

The Black-Scholes model is obviously not applicable in negative-price situations (which happened in April 2020) because the traditional Black-Scholes model assumes that all prices are positive and therefore does not cope with negative-price situations.

Oil prices are also under huge impact of political, economic, environmental, and other factors. More specifically, in the past decades, these factors include but are not limited to US spare capacity exhausted, Arab Oil Embargo, Iranian Revolution, Iran-Iraq War, the Saudis abandoning the swing producer role, Iraq invading Kuwait, the Asian financial crisis, OPEC cutting production targets 1.7 mmbpd, 9-11 attacks, Low spare capacity, Global financial collapse, OPEC cuts production targets 4.2 mmbpd, OPEC production quota unchanged, and Global pandemic reduces oil demand. [12] Prices of oil can never be accurately predicted, calculated, and modeled in the real world.

# **Discussion**

The traditional and authoritative Black-Scholes pricing model is applicable in many cases in the crude-oil field(very possibly the put option pricing model for the CME Group) whereas on the other hand, it is not applicable in a considerable amount of other causes such as in the above Indian stock case, negative-price situations, and external factors (human manipulation, political, economical, and environmental factors).

No theory or model is perfect, and neither is the Black-Scholes model. It is encouraging that the traditional Black-Scholes pricing model is still applicable in many cases and plays an essential role in nowadays oil pricing field. However, since crude oil is not a simple and ordinary product (it is often considered at the national level since it is closely related to the economy and politics of nations), its prices are affected by so many factors and sometimes do not seem to have any association with pricing models like the Black-Scholes model. These factors lie in political, economic, environmental, military, and in fact almost every aspect of nation, market, and everyday life (such as the Arab Oil Embargo, the Iranian Revolution, the Iran-Iraq War, Saudis abandoned swing producer role, global financial collapse, covid-19, etc.). For example, the market is often manipulated by humans - the Saudis said they should manipulate the market to bring the price down if they think it's not "fair" (which is \$70 to \$80). The accuracy and applicability of the B-S model in the crude oil industry always interfere with plenty of real-world factors.

In the case when B-S is obviously not a good model, some economic concepts or rules can help to integrate the pricing. For example, there is a theory saying that the "fundamental" price of oil today is equal to the total of the world's GDP today, multiplied by 3.33%, and divided by the amount of oil consumed". This method is not accurate overall but GDP can give us some insight into oil prices. Another example is "bubble" when the market prices are manipulated upwards. In this case, after the bubble pops, the prices will drop from the peak all the way to the bottom until they are much lower than the fundamental to make the bubbles zero-sum. "Reverse Bubbles" perform in similar patterns. (Andrew Butter, 2010) Besides, the Bachelier model mentioned above is a good pricing model especially when the price is down to near zero or even negative.

Further, although the limitations and inapplicability of Black-Scholes are revealed explicitly, it is extremely hard to find or establish a better pricing model for oil under uncontrollable real-world factors. Many attempts have been done to try to improve the traditional B-S model or raise new pricing models in many pieces of research but it is hard to say one of them is a universally better pricing model than Black-Scholes so far. However, a better modified and applicable "new Black-Sholes" model can be a good goal for further research investigation.

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