An Improved Nonlinear Granger Causality Test Method

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Abstract: In order to solve the problem that the traditional linear Granger causality test method cannot capture nonlinear characteristics, this paper proposes the nonlinear Granger causality test method by improving the smooth transition function of the smooth transition autoregressive (STAR) model. The empirical study examines the Granger causality between Consumer Price Index (CPI) and Producer Price Index for Industrial Products (PPI) from both linear and nonlinear perspectives, and the results show that the method has higher robustness.
Keywords: Nonlinear Granger Causality Test; Smooth Transition Autoregressive Model; Consumer Price Index; Producer Price Index For Industrial Products

1. Introduction

Granger [1] proposed the causality test to test the time-varying causal relationship between time series in economics and finance in 1969. In recent years, more and more economists, statisticians and related practitioners have paid attention to Granger causality, which means that the statistically significant Granger causality still has a very practical reference value in the real world.

Although the test method of linear Granger causality has been deeply studied [2-4], the linear Granger causality test method can hide or produce false causality for variables that actually have nonlinear Granger causality. In fact, as early as 1992, Baek and Brock [5] discovered the limitations of the linear Granger causality test, proposed a binary nonlinear Granger causality test method. Subsequently, Hiemstra and Jones [6], Diks et al. [7-8], Lee and Yang [9] continued to improve on the basis of predecessors. In order to realize the nonlinear causal relationship analysis of multivariate time series, Ren et al. [10] proposed the HSIC-Lasso-GC model, and proved the effectiveness of the proposed method through simulation.

2. Theoretical basis

2.1Linear Granger causality test

According to Granger's point of view, the vector autoregression (VAR) model is generally used to construct the causality test model. Consider two stationary time series variables X and Y to test whether the variable X granger causes the variable Y. The causality test model is

$$y_t = a_0 + \sum_{i=1}^{p} a_i y_{t-i} + \varepsilon_t^1 \#(1)$$

$$y_t = a_0 + \sum_{i=1}^{p} a_i y_{t-i} + \sum_{j=1}^{q} b_j x_{t-j} + \varepsilon_t^2 \#(2)$$

Among them, p and q are the lag orders of y_t and x_t respectively, a_0 is a constant, $a = (a_1, \cdots, a_p)'$ and $b = (b_1, \cdots, b_q)'$ are the coefficients of the model, $\varepsilon_t^1 \sim WN(0, \sigma_1^2)$ and $\varepsilon_t^2 \sim WN(0, \sigma_2^2)$ are two uncorrelated disturbance terms. If $b_j \neq 0, j = 1,2,\cdots,q$, then it is considered that X is the Granger cause of Y, otherwise, X is not the Granger cause of Y.

Note: here n is only to distinguish different disturbance terms in ε_t^{n}, the same below.

2.2 Smooth transition autoregressive model

Granger and Terӓsvirta [11] proposed the smooth transition autoregressive (STAR) model to capture nonlinear characteristics in economic activities. The general expression is

$$y_t = a_0 + \sum_{i=1}^{p} a_i y_{t-i} + \left(b_0 + \sum_{i=1}^{p} b_i y_{t-i} \right) G(s_t; \gamma, c) + \varepsilon_t^3 \#(3)$$

p is the lag order of the time series variable Y, a_0 and b_0 are constants, and $G(s_t; \gamma, c)$ is a deterministic smooth transition function, which can be either a continuous odd function or a continuous even function, $\varepsilon_t^3 \sim iid(0, \sigma_3^2)$ is the disturbance term.

3. Theoretical method

3.1 Smooth transition function

The specific form of $G(s_t; \gamma, c)$ determines how the STAR model is transitioned. With the increasing maturity and wide application of nonparametric smoothing techniques, this paper uses the kernel function as $G(s_t; \gamma, c)$ to construct the STAR model whose expression is as follows:

$$y_t = a_0 + \sum_{i=1}^{p} a_i y_{t-i} + \left(b_0 + \sum_{i=1}^{p} b_i y_{t-i} \right) K_\gamma(s_t - c) + \varepsilon_t^3 \#(4)$$

Where, $K_\gamma(\cdot) = \gamma K(\cdot/\gamma^2)$, γ is a smooth transition parameter, satisfying $\lim_{n\to\infty} \gamma_n \to 0$, $\lim_{n\to\infty} n\gamma_n \to \infty$. When $\gamma \to 0$, the STAR model degenerates into a linear model.

The specific expression of the custom kernel function $K(\cdot)$ is

$$K(x) = \frac{1}{4} \exp\left(-\frac{|x|}{2} \right) \#(5)$$

In general, the kernel function $K(x)$ has the following properties. The proof is in the appendix.

(1) Non-negativity: $K(x) \geq 0, x \in (-\infty, +\infty)$;

(2) Symmetry: $K(x) = K(-x), x \in (-\infty, +\infty)$;

(3) Normalization: $\int_{-\infty}^{+\infty} K(x)\, dx = 1$;

(4) The expected value is equal to 0, i.e. $\int_{-\infty}^{+\infty} xK(x)\, dx = 0$;

(5) There is a second-order moment, i.e. $\int_{-\infty}^{+\infty} x^2 K(x)\, dx = \sigma^2 < \infty$.

In this paper, the transition variable s_t adopts the lagged endogenous variable y_{t-d}, i.e.

$$G(s_t; \gamma, c) = K_\gamma(y_{t-d} - c) \#(6)$$

Where, y_{t-d} is the lagged endogenous variable, d is the delay parameter, and c is the threshold.

3.2 Nonlinear Granger causality test

The STAR model can capture the nonlinear characteristics of time series and help to reveal the nonlinear Granger causality between two time series variables more accurately. Therefore, this subsection constructs the nonlinear Granger causality test model based on the above-mentioned STAR model. Assuming that X and Y are two stationary time series variables. Firstly, a nonlinear test should be performed to check whether them have nonlinear characteristics. Then according to the nonlinear test situation, select the appropriate model as follows.

(1) When only X shows significant nonlinear characteristics.

$$y_t = a_0 + \sum_{i=1}^{p} a_i y_{t-i} + \varepsilon_t^1 \#(1)$$

$$y_t = a_0 + \sum_{i=1}^{p} a_i y_{t-i} + \left(b_0 + \sum_{j=1}^{q} b_j x_{t-j} \right) K_\gamma(x_{t-d} - c) + \varepsilon_t^4 \#(7)$$

(2) When only Y shows significant nonlinear characteristics.

$$y_t = a_0 + \sum_{i=1}^{p} a_i y_{t-i} + \left(b_0 + \sum_{i=1}^{p} b_i y_{t-i} \right) K_\gamma(y_{t-d} - c) + \varepsilon_t^3 \#(8)$$

$$y_t = a_0 + \sum_{i=1}^{p} a_i y_{t-i} + \left(b_0 + \sum_{i=1}^{p} b_i y_{t-i} \right) K_\gamma(y_{t-d} - c) + \sum_{j=1}^{q} c_j x_{t-j} + \varepsilon_t^5 \#(9)$$

(3) When both X and Y show significant nonlinear characteristics.

$$y_t = a_0 + \sum_{i=1}^{p} a_i y_{t-i} + \left(b_0 + \sum_{i=1}^{p} b_i y_{t-i} \right) K_\gamma(y_{t-d} - c) + \varepsilon_t^3 \#(10)$$

$$y_t = a_0 + \sum_{i=1}^{p} a_i y_{t-i} + \left(b_0 + \sum_{i=1}^{p} b_i y_{t-i} \right) K_\gamma(y_{t-d} - c) + \left(c_0 + \sum_{j=1}^{q} c_j x_{t-j} \right) K_\gamma(x_{t-d} - c) + \varepsilon_t^6 \#(11)$$

The above three nonlinear Granger causality test models are identified as constrained models and unconstrained models respectively. Then, the F-statistic is constructed based on the residual sum of squares, and the hypothesis test is carried out. Among them, when the STAR model based on variable Y is selected to test the linear effect of X on Y, even though the model tests the linear effect of X on Y, the null hypothesis at this time is still “X is not a nonlinear Granger cause of Y”, this is because the STAR model introduces the nonlinear characteristics of Y.

4. Empirical results and analysis

4.1 Linear Granger causality test results

To avoid the problem of "pseudo regression", this paper adopts ADF and PP test methods to test the stationarity of CPI and PPI of the four first-tier cities of Shanghai, Beijing, Guangzhou, and Shenzhen. Taking the CPI and PPI sequences after first-order difference processing as the sample data, and the empirical results based on the common lag order p = q = 1,2,3,4,5,6 are shown in Table 1.

Table 1 Linear Granger causality test results between CPI and PPI

lag order	H_0:CPI is not a linear Granger cause of PPI	H_0:PPI is not a linear Granger cause of CPI	conclusion
	F test statistic	F test statistic	
	Shanghai		
1	1.4674	0.0353	CPI⇎PPI
2	2.1502	0.3021	CPI⇎PPI
3	1.0686	1.0367	CPI⇎PPI
4	0.5768	0.8073	CPI⇎PPI
5	1.8203	1.8133	CPI⇎PPI
6	1.7682	1.6057	CPI⇎PPI
	Beijing		

1	0.1148	3.5147*	CPI←PPI
2	0.7021	1.9787	CPI↮PPI
3	0.7539	3.0331*	CPI←PPI
4	0.6954	2.2718*	CPI←PPI
5	0.9613	1.9153	CPI↮PPI
6	1.7175	1.6545	CPI↮PPI
Guangzhou			
1	2.7811*	0.0008	CPI→PPI
2	1.4705	0.0442	CPI↮PPI
3	2.1657	0.6373	CPI↮PPI
4	2.8398*	0.6315	CPI→PPI
5	2.8852*	1.0856	CPI→PPI
6	1.9279*	0.8743	CPI→PPI
Shenzhen			
1	0.2693	0.5106	CPI↮PPI
2	0.2602	0.1015	CPI↮PPI
3	0.4986	4.7169***	CPI←PPI
4	1.3546	4.4810***	CPI←PPI
5	1.1452	3.1443*	CPI←PPI
6	1.4532	2.8286**	CPI←PPI

From the empirical results, four first-tier cities reject the null hypothesis in different ways. Among them, when the common lag order is 1 to 6, there is no linear Granger causality between CPI and PPI in Shanghai. When the common lag order is 1, 3 and 4, Beijing significantly rejects "PPI is not a linear Granger cause of CPI" at the confidence level of 10%, indicating that there is a unidirectional linear Granger causality from PPI to CPI. However, in contrast to Beijing, when the common lag order is 1, 4, 5 and 6, Guangzhou has a unidirectional linear Granger causality from CPI to PPI. When the common lag order is 3, 4, 5 and 6, Shenzhen's F-test statistic significantly rejects "PPI is not a linear Granger cause of CPI" at the confidence level of 1%, 10% and 5%, respectively. Like Beijing, there is a unidirectional linear Granger causality from PPI to CPI.

4.2 Nonlinear Granger causality test results

In this empirical study, the BDS test method proposed by Brock et al. [12] was adopted. The results show that, except for Beijing's CPI, other time series data have nonlinear characteristics. Therefore, compared with the traditional linear Granger causality test method, the nonlinear Granger causality test method is more suitable for testing the Granger causality between CPI and PPI.

Table 2 Nonlinear Granger causality test results between CPI and PPI

City	H_0:CPI is not a nonlinear Granger cause of PPI		H_0:PPI is not a nonlinear Granger cause of CPI		conclusion
	F test value	P-value	F test value	P-value	
Shanghai	1.0470	0.3099	1.0599	0.3523	CPI↮PPI
Beijing	0.3308	0.5671	3.4188*	0.0688	CPI←PPI
Guangzhou	9.9079***	0.0026	0.7971	0.5558	CPI→PPI
Shenzhen	3.2752**	0.0439	4.6748**	0.0342	CPI↔PPI

From the empirical results in Table 2, there is no nonlinear Granger causality between CPI and PPI in Shanghai. Beijing significantly rejects “PPI is not a nonlinear Granger cause of CPI” at the significance level of 10%, indicating that there is a unidirectional nonlinear Granger causality from PPI to CPI. However, Guangzhou significantly rejects “CPI is not a nonlinear Granger cause of PPI” at the significance level of 1%, indicating that there is a unidirectional nonlinear Granger causality from CPI to PPI in Guangzhou. Shenzhen's F-test statistic significantly rejects “CPI is not a nonlinear Granger cause of PPI” and “PPI is not a nonlinear Granger cause of CPI” at the significance level of 5%, indicating that there is a bidirectional nonlinear Granger causality between CPI and PPI in Shenzhen.

4.3 Comparative analysis and discussion

Table3 Linear and nonlinear Granger causality test results between CPI and PPI

City	Linear Granger causality test results	Nonlinear Granger causality test results
Shanghai	CPI↮PPI	CPI↮PPI
Beijing	CPI←PPI	CPI←PPI
Guangzhou	CPI→PPI	CPI→PPI
Shenzhen	CPI←PPI	CPI↔PPI

From linear perspective, there is no Granger causality between CPI and PPI in Shanghai, which may be because the time series itself has nonlinear characteristics, and the traditional linear Granger causality test method cannot test. PPI in Beijing and Shenzhen is a Granger cause of CPI changes, which means that changes in supply factors cause fluctuations in price levels to a large extent, which may lead to the risk of “cost-push inflation”. The CPI in Guangzhou is a Granger cause of PPI changes, which means that the demand factor has become the dominant factor causing price level fluctuations, and there is likely to be the risk of "demand-pull inflation".

From nonlinear perspective, even though both CPI and PPI in Shanghai have nonlinear characteristics, there is no Granger causality between them, which indicates that CPI and PPI in Shanghai have little mutual influence. PPI in Beijing is Granger causality that causes CPI changes, which means that the price changes of production materials and other living materials, may be transmitted to the downstream consumer commodity price. Resulting in the fluctuation of price level, which is in line with the theory of “production chain transmission”, and PPI can be used as an important macroeconomic indicator to predict “cost-push inflation”. However, the situation in Guangzhou is opposite to that in Beijing, which may be because the raw materials required to produce some products in PPI are just consumer goods in CPI. The price of raw materials changes with the changes in consumption demand factors, which in turn causes changes in the prices of upstream production products, which is in line with the “induced demand” theory. Therefore, CPI can be used as an important indicator to predict “demand-pull inflation”. CPI and PPI in Shenzhen are each other's nonlinear Granger reasons, which shows that above two theories can be established at the same time.

5. Conclusion

In this paper, the smooth transition function in the smooth transition autoregressive model is improved, and the nonlinear Granger causality test method is proposed. The Granger causality between CPI and PPI in first-tier cities is explored from both linear and nonlinear perspectives. The empirical results show that the traditional linear Granger causality test method hides the real causality because it ignores the nonlinear characteristics, and the nonlinear Granger causality test method proposed can draw more robust conclusions. The results can help government workers in first-tier cities to clarify the relationship between CPI and PPI, identify the type of inflation, and adopt policies to control inflation to macro-control the price level as soon as possible.

Appendix

Obviously, the kernel function $K(x)=\frac{1}{4}\exp\left(-\frac{|x|}{2}\right)$ has non-negativity and symmetry, and only proofs of other properties are given below.

$$\int_{-\infty}^{+\infty}\frac{1}{4}\exp\left(-\frac{|x|}{2}\right)dx=\frac{1}{2}\int_{0}^{+\infty}\exp\left(-\frac{x}{2}\right)dx=-\exp\left(-\frac{x}{2}\right)\Big|_{0}^{+\infty}=1\#(12)$$

It is easy to obtain that the integral of the kernel function $K(x)$ on $(-\infty, +\infty)$ is equal to 1, which satisfies the normalization.

$$\int_{-\infty}^{+\infty} \frac{x}{4} \exp\left(-\frac{|x|}{2}\right) dx = 0\#(13)$$

Obviously, the expected value of the kernel function $K(x)$ is equal to 0.

$$\int_{-\infty}^{+\infty} \frac{x^2}{4} \exp\left(-\frac{|x|}{2}\right) dx = \frac{1}{2}\int_{0}^{+\infty} x^2 \exp\left(-\frac{x}{2}\right) dx = -\int_{0}^{+\infty} x^2 de^{-\frac{x}{2}} = -\left(x^2 e^{-\frac{x}{2}}\Big|_0^{+\infty} - 2\int_{0}^{+\infty} xe^{-\frac{x}{2}} dx\right) = -4\int_{0}^{+\infty} xde^{-\frac{x}{2}}$$

$$= -4\left(xe^{-\frac{x}{2}}\Big|_0^{+\infty} - \int_{0}^{+\infty} e^{-\frac{x}{2}} dx\right) = -8e^{-\frac{x}{2}}\Big|_0^{+\infty} = 8$$

(14)

It follows from this that the second-order moment σ^2 of the kernel function $K(x)$ is equal to 8, and there is obviously a second-order moment.

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